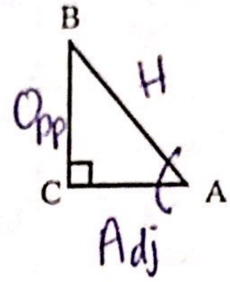
Right Triangle Trigonometry *degree mode*

For an acute angle A in right triangle ABC, the trigonometric functions are as follow:

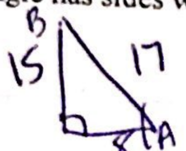
sin A = $\frac{opp}{hyp}$

cos A = $\frac{adj}{hyp}$

tan A = $\frac{opp}{adj}$



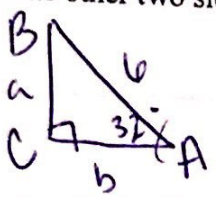
1. A right triangle has sides whose lengths are 8 cm, 15 cm, and 17 cm. Find the values of missing angles.



$\sin A = \frac{15}{17}$ cosA

$A = 61.93°$ $B = 28.07°$

2. A right triangle with a hypotenuse of 6 includes a 32° angle. Find the measure of the other two angles and the lengths of the other two sides.



$B = 58°$
$C = 90°$

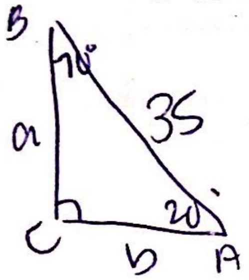
$6 \cdot \sin 32 = \frac{a}{\cancel{6}} \cdot \cancel{6}$ $6 \cdot \cos 32 = \frac{b}{\cancel{6}} \cdot \cancel{6}$

$a = 3.18$ $b = 5.09$

3. Solve each right triangle. Round angles and sides to the nearest hundredth. $\angle C = 90°$

To solve a triangle, means to find all missing sides + angles.

a. A = 20°, c = 35

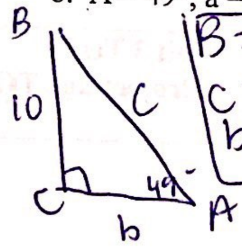


$\boxed{B = 70°,\ a = 11.97,\ b = 32.89}$

$\sin 20 = \frac{a}{35}$

$\cos 20 = \frac{b}{35}$

b. A = 49°, a = 10



$\boxed{B = 41°,\ c = 13.25,\ b = 8.69}$

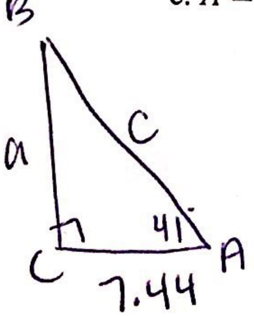
$\sin 49 = \frac{10}{c}$

$c = \frac{10}{\sin 49}$

$\tan 49 = \frac{10}{b}$

$b = \frac{10}{\tan 49}$

c. $A = 41°, b = 7.44$

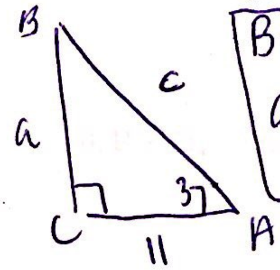


$\boxed{B = 49°,\ a = 6.47,\ c = 9.86}$

$\cos 41 = \frac{7.44}{c}$

$\tan 41 = \frac{a}{7.44}$

d. $A = 37°, b = 11$

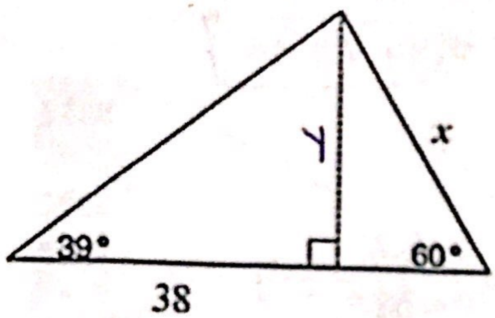


$\boxed{B = 53°,\ a = 8.29,\ c = 13.77}$

$\cos 37 = \frac{11}{c}$

$\tan 37 = \frac{a}{11}$

4. Solve for x.



$38 \cdot \tan 39 = \frac{y}{38} \cdot 38$

$y = 30.77$

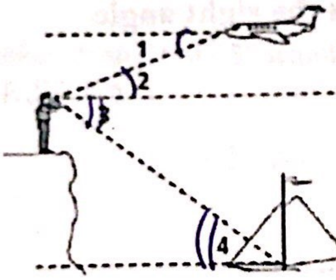
$\sin 60 = \frac{y}{x}$

$x = \frac{y}{\sin 60}$

$\boxed{x = 35.53}$

Angle of Elevation

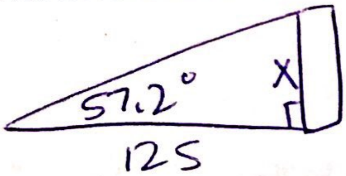
From horizontal looking up



Angle of Depression

From horizontal looking down

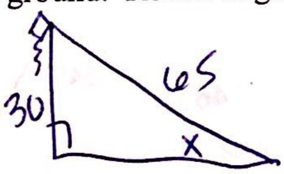
5. From a point on level ground 125 feet from the base of a tower, the angle of elevation is 57.2°. Approximate the height of the tower to the nearest foot.



$\tan 57.2 = \frac{x}{125}$

$x = 193.96$ ft

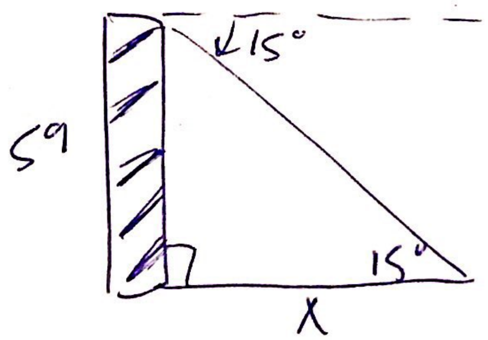
6. A kite flies at a height of 30 feet when 65 feet of string is out. If the string is in a straight line, find the angle that it makes with the ground. Round angle to the nearest tenth.



$\sin x = \frac{30}{65}$

$x = \sin^{-1}\left(\frac{30}{65}\right) = 27.49°$

7. A ship is spotted from a lighthouse at Cape Hatteras at an angle of depression of 15^0. How far from Cape Hatteras is the ship if the lighthouse is 59 m high?



$\tan 15° = \frac{59}{x}$

$x = \frac{59}{\tan 15°} = 220.19$ m

8. From an apartment window 24 m above the ground, the angle of depression of the base of a nearby building is 38^0 and the angle of elevation of the top is 63^0. Find the height of the nearby building (to the nearest tenth of a meter).

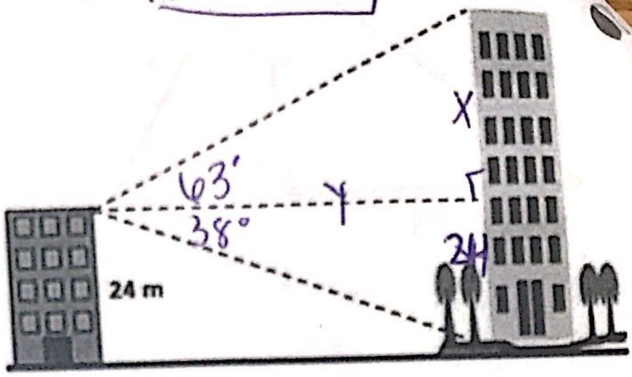
③ $60.29 + 24 = \boxed{84.29\ m}$

① $\tan 38 = \frac{24}{y}$

$y = \frac{24}{\tan 38} = 30.72$

② $\tan 63 = \frac{x}{30.72}$

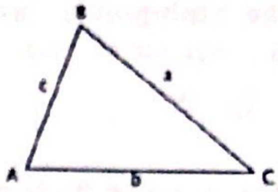
$x = 60.29$



Practice - Solve each right triangle. Round lengths to two decimal places and angles to one decimal place. Assume angle C is the right angle.

Day 2 Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



*Used when **not** a right triangle and you have **ASA or AAS** from geometry

***Make sure you are in Degree mode!!!!

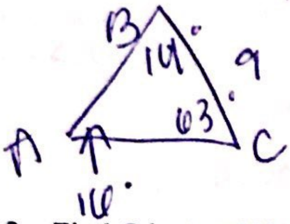
Important notes:

*Side lengths are represented with lowercase letters

*Angle measures are represented with Capital letters

*The largest side is opposite the largest angle

1. Find c if B = 101°, C = 63°, and a = 9

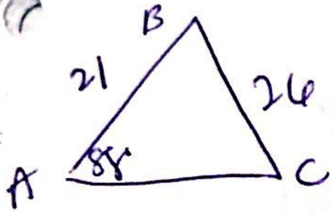


$$\frac{\sin 16}{9} \times \frac{\sin 63}{c}$$

$$\frac{9\sin 63}{\sin 16} = c\sin 16$$

$\boxed{c = 29.09}$

2. Find C is A = 88°, a = 26, c = 21

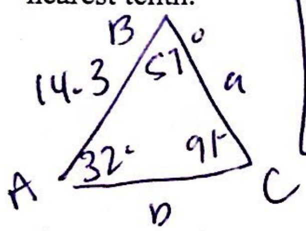


$$\frac{\sin C}{21} \times \frac{\sin 88}{26}$$

$$26\sin C = \frac{21\sin 88}{26}$$

$\boxed{C = 53.82°}$

3. **Solve** triangle ABC if $A = 32°$, $B = 57°$, and c=14.3. Round angle measures and side measures to the nearest tenth.



$\boxed{\begin{array}{l} C = 91° \\ a = 7.58 \\ b = 11.99 \end{array}}$

$$\frac{\sin 32}{a} = \frac{\sin 91}{14.3}$$

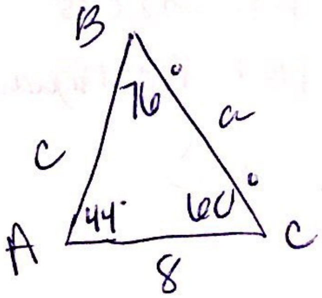
$$\frac{a\sin 91}{\sin 91} = \frac{14.3\sin 32}{\sin 91}$$

$\boxed{a = 7.58}$

$$\frac{\sin 91}{14.3} = \frac{\sin 57}{b}$$

$$b = \frac{14.3\sin 57}{\sin 91}$$

4. Practice: Solve triangle ABC if B = 76°, C = 60°, and b = 8.



$\boxed{\begin{array}{l} A = 44° \\ a = 5.73 \\ c = 7.14 \end{array}}$

$$\frac{\sin 76}{8} = \frac{\sin 44}{a}$$

$$a = \frac{8\sin 44}{\sin 76}$$

$$\frac{\sin 76}{8} = \frac{\sin 60}{c}$$

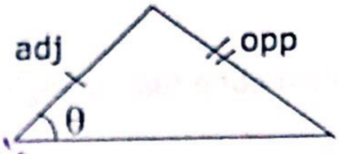
$$c = \frac{8\sin 60}{\sin 76}$$

The Ambiguous Case (SSA)

SSA does **not** guarantee a unique triangle (or even that a Δ exists). Therefore SSA is called the

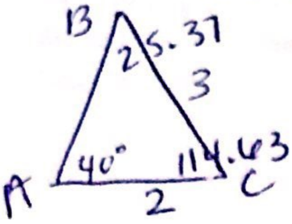
Ambiguous case.

SSA means 2 sides and a non-included angle



∢ 180 − ∢ found = # + Given angle = # > 180, 1 Δ
< 180, 2 Δ's

Example 1: Solve the triangle: a = 3, b = 2, A = 40°.



$\frac{\sin 40}{3} = \frac{\sin B}{2}$

$\sin B = \frac{2\sin 40}{3}$

$\boxed{B = 25.37°}$

$\boxed{C = 114.63°}$

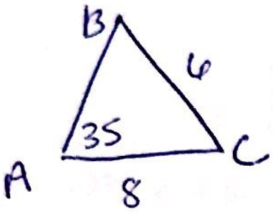
$\frac{\sin 114.63}{c} = \frac{\sin 40}{3}$

180 − 25.37 = 154.63
+ 40
194.63°

1 Δ only!

$\boxed{c = 4.24}$

Example 2: Solve the triangle: a = 6, b = 8, A = 35°.



$\frac{\sin 35}{6} = \frac{\sin B}{8}$

$\boxed{\begin{array}{l} B_1 = 49.89° \\ C_1 = 95.11° \end{array}}$

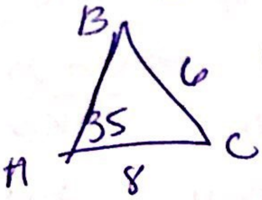
$\frac{\sin 95.11}{c} = \frac{\sin 35}{6}$

$\boxed{c_1 = 10.42}$

180 − 49.89 = 130.11
+ 35
165.11

2 Δ's!

2nd Δ

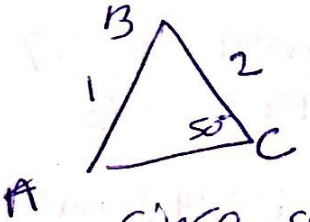


$\boxed{\begin{array}{l} B_2 = 130.11° \\ C_2 = 14.89° \end{array}}$

$\frac{\sin 14.89}{c} = \frac{\sin 35}{6}$

$\boxed{c = 2.69}$

Example 3: Solve the triangle: a = 2, c = 1, C = 50°.



$\frac{\sin 50}{1} = \frac{\sin A}{2}$

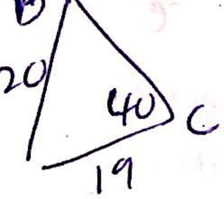
$\sin A = \frac{2\sin 50}{1}$

A = error

No Δ!

Draw a picture and then determine the number of triangles.

1. C=40°, b=19, c=20



$\frac{\sin 40}{20} = \frac{\sin B}{19}$

B = 37.64

180 − 37.64 = 142.36
+ 40
182

1 Δ

2. B=67°, A=13°, b=22

~~Sin~~

Not Ambiguous

1 Δ

3. B=23°, a=9.2, C=65°

not Ambiguous

1 Δ

$2\Delta's$

4. B=45°, a=28, b=20

$\frac{\sin 45}{20} = \frac{\sin A}{28}$

$A = 81.87°$

$180 - 81.87 = 98.13 + 45$ $\overline{143}$

$2\Delta's$

5. C=27°, a=21, c=15

$\frac{\sin 27}{15} = \frac{\sin A}{21}$

$A = 39.46$

$180 - 39.46 = 140.54$ $\underline{+27}$ 167.54

6. A=40°, a=10, c=18

$\frac{\sin 40}{10} = \frac{\sin C}{18}$

C = err

NO Δ

7. A=36°, a=19, b=17

$\frac{\sin 36}{19} = \frac{\sin B}{17}$ 1Δ

$B = 31.73$

$180 - 31.73 = 148.27 + 36 = 184.27$

8. B=112°, a=12, b=11

No Δ

$\frac{\sin 112}{11} = \frac{\sin A}{12}$

A = err

9. A=27°, b=25, C=26°

$\frac{\sin 27}{}$

1Δ

not ambiguous

Practice - Law of Sines

A. Find the indicated measurement. **(angles to the nearest tenth)**

1. Find b if A=118°, B=22°, c=24

2. Find C if A=88°, a=26, b=16.1, c=21

3. Find c if B=44°, C=53°, b=7

4. Find C if A=82°, a=20, b=24, c=20

5. Find a if A=39°, C=51°, b=27

6. Find C if A=103°, a=26, b=24, c=6

7. Find c if B=101°, C=63°, a=9

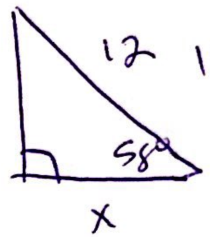
8. Find A if B=129°, a=11, b=33, c=25

Applications of Trig, Law of Sines & Law of Cosines

*Draw a picture!

*If it's a right triangle, will use trig at some point

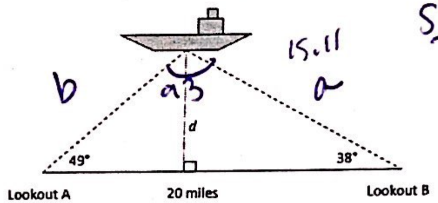
A 12 foot ladder is leaned against a house. The angle of elevation from the ground to the house is 58º. How far from the house is the ladder placed?



$12 \cos 58 = \frac{x}{12} \cdot 12$

$x = 6.36 \text{ ft}$

Triangulation can be used to find the location of an object by measuring the angles to the object from two points at the end of a baseline. Two lookouts 20 miles apart on the coast spot a ship at sea. Using the figure below find the distance, d, the ship is from shore to the nearest tenth of a mile.



$\frac{\sin 49}{a} = \frac{\sin 93}{20}$

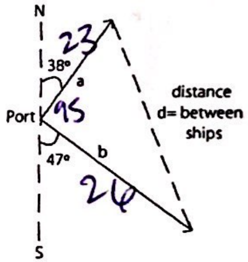
$a = \frac{20 \sin 49}{\sin 93}$

$a = 15.11$

$\sin 38 = \frac{d}{15.11}$

$\boxed{d = 9.306 \text{ mi}}$

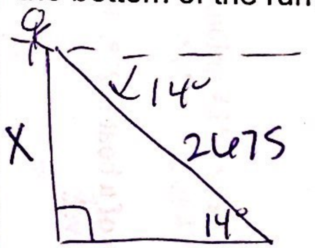
Two ships leave port at 4 p.m. One is headed at a bearing of N 38 E and is traveling at 11.5 miles per hour. The other is traveling 13 miles per hour at a bearing of S 47 E. How far apart are they when dinner is served at 6 p.m.?



$d^2 = 23^2 + 26^2 - 2(23)(26)\cos 95$

$d = 36.18 \text{ miles}$

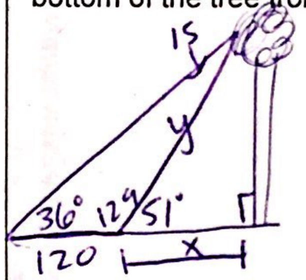
Dean was skiing at Sugar Mountain last week in Boone, NC. He was at the top of Whoopdeedoo. He measured the angle of depression to the bottom of the run to be 14º. He read that the actual length of the run was 2675 feet. What is the change in altitude to the bottom of the run?



$\sin 14 = \frac{X}{2675}$

$X = 647.14$ ft

Yogi bear is out walking with Boo Boo looking for a picnic basket and Boo Boo sights a bee hive up in a tree at an angle of elevation of 36º. They walk 120 meters closer to get a better look and then look up at an angle of elevation of 51º. How far are Yogi and Boo Boo from the bottom of the tree from their current spot?



$\frac{\sin 15}{120} = \frac{\sin 36}{y}$

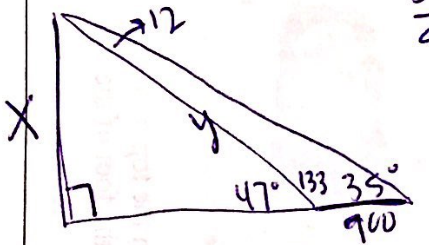
$y = \frac{120 \sin 36}{\sin 15}$

$y = 272.52$

$\cos 51 = \frac{X}{272.52}$

$\boxed{X = 171.50 \text{ ft}}$

To measure the height of the mountain, a surveyor takes two sightings of the peak at a distance 900 m apart on a direct line to the mountain. The first observation results in an angle of elevation of 35º and the second results in an angle of elevation of 47º. If the surveyor is 2 meters high, how tall is the mountain?



$\frac{\sin 12}{900} = \frac{\sin 35}{y}$

$y = \frac{900 \sin 35}{\sin 12}$

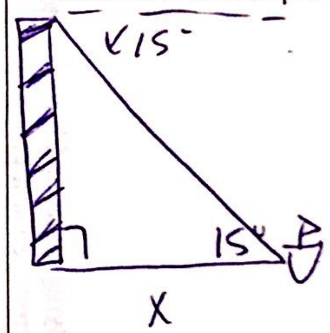
$y = 2482.88$

$\sin 47 = \frac{x}{2482.88}$

$X = 1815.86 + 2 = \boxed{1817.86 \text{ m}}$

59

A ship is spotted from a lighthouse at Cape Hatteras at an angle of depression of 15º. How far from Cape Hatteras is the ship if the lighthouse is 59 m high?



$\tan 15 = \frac{59}{X}$

$X = \frac{59}{\tan 15}$

$\boxed{X = 220.20}$