Unit 6 Day 1 – Counting Principle Name ______________

Vocabulary:

*Outcome - the result of an experiment or single trial

*Sample Space - the set of all possible outcomes

*Event - the set of outcomes that satisfy a condition

*Independent Events - events that do not affect each other

*Dependent Events - events that do affect each other

Make a tree diagram to show all the possible outcomes for the following problem.

1. A sandwich cart offers customers a choice of hamburger, chicken, or fish on either a plain or a sesame seed bun. How many different combinations of meat and a bun are possible?

H < P, S
C < P, S
F < P, S

6 combos

2. Determine if the following events are independent or dependent.

a. You spin a spinner and roll a dice I

b. A jar has 5 red marbles and 3 blue. You draw a marble, replace it, then draw another. I

c. You have a pile of Scrabble tiles. You draw the letter M and then draw the letter E without replacement. D

***Fundamental Counting Principle:**

If there are <u>m</u> ways to make 1st selection + <u>n</u> ways to make a 2nd selection, then there are <u>m·n</u> ways to make both selections

3. Kim won a contest on a radio station. The prize was a restaurant gift certificate and tickets to a sporting event. She can select one of the three different restaurants and tickets to a football, baseball, basketball, or hockey game. How many different ways can she select a restaurant followed by a sporting event?

$3(4) = 12$ ways

4. For their vacation, the Murray family is choosing a trip to the beach or the mountains. They can select their transportation from a car, plane, or train. How many different ways can they select a destination followed by a means of transportation?

$2(3) = 6$ ways

5. A bookshelf holds 4 different biographies and 5 different mystery novels. How many ways can one book of each type be selected?

$4(5) = 20$ ways

6. For a college application, Ophelia must select one of five topics on which to write a short essay. She must also select a different topic from the list for a longer essay. How many ways can she choose the topics for the two essays?

$5(4) = 20$ ways

7. How many 3 letter, 3 digit license plates are there if the letters cannot be repeated but the numbers can?

$\underset{L}{\underline{26}}\ \underset{L}{\underline{25}}\ \underset{L}{\underline{24}}\ \underset{\#}{\underline{10}}\ \underset{\#}{\underline{10}}\ \underset{\#}{\underline{10}} = 15,600,000$

6

8. If you flip a penny, nickel, and dime, how many different outcomes are there?

$\frac{2}{P} \cdot \frac{2}{N} \cdot \frac{2}{D} = 8$

9. How many seven-digit phone numbers are there?

$10^7 = 10,000,000$

10. How many seven-digit phone numbers can begin with 28?

$\frac{1}{2} \cdot \frac{1}{8} \cdot \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 100,000$

10^5

11. If you have an 8 question multiple choice quiz, where there are 3 answer choices for each question, how many different ways could the quiz be answered?

$\underline{3}\ \underline{3}\ \underline{3}\ \underline{3}\ \underline{3}\ \underline{3}\ \underline{3}\ \underline{3} = 6561$

3^8

12. If there are 3 people running a race, how many different ways exist for them to place?

$\underset{1st}{\underline{3}}\ \underset{2nd}{\underline{2}}\ \underset{3rd}{\underline{1}} = 6$

2

Introduction to Factorials

Factorial: the product $n(n-1)(n-2)\ldots 1$ is called <u>n factorial</u> and is symbolized by **n!**

13. $0! = 1$ 0 objects, how many ways can you make this?

14. Evaluate each expression

a. $5!$

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 120$

b. $10!$

$\cancel{20 \cdot 19 \cdot 18 \cdot 17} \ldots$

$10 \cdot 9 \cdot 8 \cdot 7 \ldots$

$= 3{,}628{,}800$

c. $\frac{10!}{3!6!}$

$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{3!\, \cancel{6!}}$

$= \frac{10 \cdot 9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = \frac{5040}{6} = \boxed{840}$

d. $\frac{3!}{6!-3!}$ $6 \cdot 5 \cdot 4 \cdot \cancel{3!}$

$\frac{\cancel{3!}}{\cancel{3!}(120-1)} = \frac{1}{119}$

e. $\frac{n!}{(n+1)!}$

$\frac{\cancel{n!}}{(n+1)\cancel{(n)!}} = \frac{1}{n+1}$

f. $\frac{9!}{0!}$

$\frac{9!}{1} = 9!$

$= 362{,}880$

g. $\frac{3!+6!}{6!-3!}$

$\frac{3!(1+120)}{3!(120-1)} = \frac{121}{119}$

h. $\frac{(x+1)!(x-2)!}{(x-4)!(x-1)!}$

$\frac{(x+1)(x)\cancel{(x-1)!}\,(x-2)(x-3)\cancel{(x-4)!}}{\cancel{(x-4)!}\,\cancel{(x-1)!}}$

$= x(x+1)(x-2)(x-3)$

i. $\frac{(3n+2)!}{(3n+3)!}$

$\frac{\cancel{(3n+2)!}}{(3n+3)\cancel{(3n+2)!}} = \frac{1}{3n+3}$

j. $\frac{(2n+1)!}{(2n+3)!}$

$\frac{\cancel{(2n+1)!}}{(2n+3)(2n+2)\cancel{(2n+1)!}}$

$= \frac{1}{(2n+3)(2n+2)}$

Unit 6 Day 2 – Permutations

Name ____________________

Permutations: The arrangement of objects in a certain order where the order of the objects <u>MATTER!</u>

$_nP_n = P(n,n) = n!$ n objects taken n at a time (all objects are chosen or used)

$_nP_r = P(n,r) = \dfrac{n!}{(n-r)!}$ n objects taken r at a time (not all objects are chosen or used)

1. Evaluate:

a) P(8,2)

$_8P_2 = \dfrac{8!}{(8-2)!} = \dfrac{8!}{6!} = 56$

b) P(9,1)

$_9P_1 = \dfrac{9!}{(9-1)!} = \dfrac{9!}{8!} = 9$

c) P(7,5)

$_7P_5 = \dfrac{7!}{(7-5)!} = \dfrac{7!}{2!} = 2520$

Find the following. Show all necessary work.

2. How many ways can five different textbooks be displayed in a row on a shelf?

$_5P_5 = 5! = 120$ or $\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$

3. At the 2000 Olympic Games, eight women qualified for the women's 400-meter finals in track and field. Only three of the women can win medals. How many different ways could the top three medal winners occur?

$_8P_3 = \dfrac{8!}{5!} = 336$ or $\underline{8} \cdot \underline{7} \cdot \underline{6}$

4. Suppose you have 10 books and you only want to arrange 3 of them on a shelf. How many different ways can it be done?

$_{10}P_3 = \dfrac{10!}{7!}$ or $10 \cdot 9 \cdot 8 = 720$

5. How many different ways can you arrange a group of four people in a straight line for a picture (everyone wants to be in the picture)?

$_4P_4 = 4!$ $\quad 4 \cdot 3 \cdot 2 \cdot 1 = 24$

6. Suppose you have 6 people and you want only 4 of them at a time to be in your picture. How many different pictures can you take?

$_6P_4 = \dfrac{6!}{2!}$ or $6 \cdot 5 \cdot 4 \cdot 3 = 360$

7. Find the number of different ways the letters of the word GUITAR can be arranged.

$6! = 720$

8. How many ways can the letters in GUITAR be arranged if the first letter must be a T?

$\underset{T}{\underline{1}} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 120$

6

9. How many ways can the letters in GUITAR be arranged if the first letter must be a vowel?

$\underline{3} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 360$

10. How many ways can the letters in GUITAR be arranged if the first letter cannot be a vowel?

$\underline{3} \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 360$

11. How many ways can 10 juniors run for the positions of president, vice president, secretary, and treasurer?

$_{10}P_4 = \frac{10!}{6!}$ or $10 \cdot 9 \cdot 8 \cdot 7 = 5040$

12. How many different ways can you arrange the letters in GEOMETRY? (be careful, since there are 2 E's)

Permutations with repetitions: n = total objects, p = # alike objects, q = # alike objects

FORMULA: $\frac{n!}{p!q!}$

13. How many different ways can the letters of the word MISSISSIPPI be arranged?

$\frac{11!}{4!4!2!} = 34,650$

4 S's 4 I's
2 Ps

14. How many different ways can the letters of the word MATHEMATICS be arranged?

$\frac{11!}{2!2!2!} = 4,989,600$

2 Ms
2 A's
2 Ts

15. How many different ways can the letters of the word PROBABILITY be arranged?

$\frac{11!}{2!2!} = 9,979,200$

2 B's
2 I's

7

Scanned with CamScanner

Unit 6 Day 3 – Combinations

Name ______________________

Combination: an arrangement of objects in which order **DOES NOT MATTER!**

Combination Formula: The number of combinations of n objects taken r at a time, is written C(n, r).

$$C(n,r) = {}_nC_r = \frac{n!}{(n-r)!\,r!}$$

only difference

Evaluate:

a) $C(8,8)$

$_8C_8 = \frac{8!}{1!\,8!} = 1$

b) $P(8,3) \cdot C(9,6) \cdot C(8,2)$

$\frac{8!}{5!} \cdot \frac{9!}{3!\,6!} \cdot \frac{8!}{6!\,2!} = 336(84)(28) = 790{,}272$

1. There are 4 students on student council that want to be on the recycling committee. How many different groups can be chosen when the recycling committee has 3 openings?

$_4C_3 = \frac{4!}{(4-3)!\,3!} = \frac{4!}{1!\,3!} = 4$

2. A group of seven students working on a project needs to choose two from their group to present the group's report to the class. How many ways can they chose the two students?

$_7C_2 = \frac{7!}{(7-2)!\,2!} = \frac{7!}{5!\,2!} = 21$

3. Five cousins at a family reunion decide that three of them will go to pick up a pizza. How many ways can they choose the three people to go?

$_5C_3 = \frac{5!}{2!\,3!} = 10$

4. From a row of eight different varieties of soup on a shelf, how many groups of five cans can be selected?

$_8C_5 = \frac{8!}{3!\,5!} = 56$

5. An ice cream shop has eight different toppings from which to choose. This week, if you buy three toppings for your sundae, you get another topping free. How many different ways can the sundae be made (with 4 toppings)?

$_8C_4 = \frac{8!}{4!\,4!} = 70$

9

6. From a g

6. From a group of 6 men and 4 women, how many committees of 2 men and 2 women can be formed?

$$\underbrace{{}_6C_2}_{\text{men}} \cdot \underbrace{{}_4C_2}_{\text{women}} = \frac{6!}{4!2!} \cdot \frac{4!}{2!2!} = 15 \cdot 6 = 90$$

7. A bag contains 6 green, 5 yellow, and 8 white marbles. How many ways can 2 green, 1 yellow, and 4 white marbles be chosen?

$$\underbrace{{}_6C_2}_{\text{Green}} \cdot \underbrace{{}_5C_1}_{\text{yellow}} \cdot \underbrace{{}_8C_4}_{\text{white}} = \overbrace{\frac{6!}{4!2!}}^{15} \cdot \overbrace{\frac{5!}{4!1!}}^{5} \cdot \overbrace{\frac{8!}{4!4!}}^{70} = 5250$$

8. A bag contains 4 green, 3 yellow, and 6 white marbles. How many ways can 3 of one color and 2 on another color be chosen?

9. Five cards are drawn from a standard deck of cards. How many hands consist of three clubs and two diamonds?

$$\underbrace{{}_{13}C_3}_{\text{Clubs}} \cdot \underbrace{{}_{13}C_2}_{\text{Diamonds}} = \overbrace{\frac{13!}{10!3!}}^{286} \cdot \overbrace{\frac{13!}{11!2!}}^{78} = 22308$$

10. Six cards are drawn from a standard deck of cards. How many hands consist of two hearts and four spades?

$$\underbrace{{}_{13}C_2}_{\text{Hearts}} \cdot \underbrace{{}_{13}C_4}_{\text{Spades}} = \overbrace{\frac{13!}{11!2!}}^{78} \cdot \overbrace{\frac{13!}{9!4!}}^{715} = 55,770$$

Permutation (**ORDER IS IMPORTANT**) or Combination (**ORDER NOT IMPORTANT**)? **Then solve.**

P / C - 11. Sally has 7 candles, each a different color. How many ways can she arrange the candles in a candelabra that holds 3 candles?

P / C - 12. 50 people buy raffle tickets. 3 winners each getting $500 are selected. In how many ways can prizes be awarded?

P / C - 13. Baskin Robins offers 31 flavors of ice cream. One item they offer is a bowl with 3 scoops, each a different flavor. How many bowls are possible?

10

Unit 6 Day 4 – Multiplying Probability

Name ______________________

Definition of Probability: $\frac{\text{desired outcomes}}{\text{total possible outcomes}}$

*Probability will always be between 0 & 1.

*Probabilities should be written as fractions (simplified).

P(event occurring) = probability of an event occurring

P(event NOT occurring) = 1 − P(event occurring)

$P(A) + P(A^c) =$ 1

1. A box contains 4 basketballs, 3 footballs and 15 volleyballs. What is the probability that a volleyball will be chosen?

$P(\text{volleyball}) = \frac{15}{22}$

2. What is the probability that a volleyball will NOT be chosen?

$P(\text{no volleyball}) = \frac{7}{22}$

3. Two cards are drawn at random from a standard deck of 52 cards. Find P(both hearts).

$P(\heartsuit, \heartsuit) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$

4. Three cards are drawn at random from a standard deck of 52 cards. Find P(all 3 are clubs).

$P(C, C, C) = \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = \frac{11}{850}$

INDEPENDENT EVENTS: events whose outcomes are not affected by previous outcomes

If A and B are independent, then the probability of both events occurring is: P(A and B)= $P(A) \cdot P(B)$

5. Find the probability of rolling a 5 on the first toss of 2 dice and a 1 on the second toss.

$P(5, 1) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

6. Three dice are rolled. Find the probability the first 2 rolls are both 6 and the third roll is not a 6.

$\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{5}{216}$

7. You have 9 dimes and 7 pennies in a bag. You select one coin, then put it back and randomly select another. Find P(both dimes).

$\frac{9}{16} \cdot \frac{9}{16} = \frac{81}{256}$

DEPENDENT EVENTS: events whose outcomes are affected by previous outcomes

If A and B are dependent, then the probability of both events occurring is: P(A and B) = P(A) · P(B after A)

8. Kyle has 4 navy socks and 6 black socks in a drawer. One dark morning he randomly pulls out 2 socks. What is the probability that he will select a pair of navy socks?

$P(N, N) = \frac{4}{10} \cdot \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$

$\frac{_{16}C_4 \cdot {_{10}C_4}}{_{26}C_8}$ $\frac{8!}{4!4!}$ → 70

– 9. Cameron has a collection of 16 fiction books and 10 nonfiction books. She randomly chooses 8 books. Find the probability she chooses 4 fiction and 4 nonfiction.

$P(F,F,F,F,N,N,N,N) = \frac{16}{26}\,\frac{15}{25}\,\frac{14}{24}\,\frac{13}{23}\,\frac{10}{22}\,\frac{9}{21}\,\frac{8}{20}\,\frac{7}{19} = \left(\frac{28}{3795}\right)^{70} = \frac{1176}{4807}$

10. A collection of 38 computer disks contains four that are defective. If two are selected at random, what is the probability that at least one of them is good?

$P(G, B) = \frac{34}{38} \cdot \frac{4}{37} = \frac{68}{703}$

$P(G, G) = \frac{34}{38} \cdot \frac{33}{37} = \frac{561}{703}$

$P(B, G) = \frac{68}{703}$ Add together $= \frac{697}{703}$

OR

$1 - P(B, B)$

$= 1 - \left(\frac{4}{38} \cdot \frac{3}{37}\right)$

$= \frac{697}{703}$

11. The host of a game show is drawing slips of paper with prizes. Of the 10 slips, 6 show TV, 3 show vacation, and 1 shows car. If the host draws the slips at random and does not replace them, find each.

__________ a. P(vacation, then a car)

$\frac{3}{10} \cdot \frac{1}{9} = \frac{1}{30}$

__________ b. P(2 TV'S)

$\frac{6}{10} \cdot \frac{5}{9} = \frac{1}{3}$

__________ c. P(vacation and TV)

VT + TV

$\frac{3}{10} \cdot \frac{6}{9} + \frac{6}{10} \cdot \frac{3}{9}$

$= 2/5$

0 d. P(2 cars)

$\frac{1}{10} \cdot \frac{0}{9}$

Assume **without replacement** if it does not specify.

12. Two cards are drawn at random from a standard deck of 52 cards. What is the probability that both cards are spades?

$P(S,S) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$

13. Three cards are drawn from a standard deck of cards. Find the probability of drawing a diamond, a club, and another diamond **in that order.**

$P(D,C,D) = \frac{13}{52} \cdot \frac{13}{51} \cdot \frac{12}{50} = \frac{13}{850}$

14. A box contains 6 red markers, 3 green markers, and 4 blue markers. Three are selected at random.

13 total

a. P(3 Red)

$\frac{6}{13} \cdot \frac{5}{12} \cdot \frac{4}{11} = \frac{10}{143}$

b. P(1 of each) RBG RGB BRG GRB GBR BGR 6 ways

$6\left(\frac{6}{13} \cdot \frac{3}{12} \cdot \frac{4}{11}\right) = \frac{36}{143}$

c. P(2 Red, 1 Blue) RRB RBR BRR

$3\left(\frac{6}{13} \frac{5}{12} \frac{4}{11}\right) = \frac{30}{143}$

d. P(not green)

$\frac{10}{13} \cdot \frac{9}{12} \cdot \frac{8}{11} = \frac{60}{143}$

~~1-P(3G)~~

$\cancel{1-\left(\frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10}\right) = \frac{219}{220}}$

Practice with Probability

1. Ebony has 4 male kittens and 7 female kittens. She picks up 2 kittens to give to a friend. Find the probability of each selection.

a. P(2 male)

$\frac{4}{11} \cdot \frac{3}{10} = \frac{6}{55}$

b. P(2 female)

$\frac{7}{11} \cdot \frac{6}{10} = \frac{21}{55}$

c. P(1 of each) MF FM

$2\left(\frac{4}{11} \cdot \frac{7}{10}\right) = \frac{28}{55}$

2. Bob is moving and all of his CDs are mixed up in a box. Twelve CDs are rock, eight are jazz, and five are classical. If he reaches in the box and selects them at random, find each probability.

a. P(3 jazz)

b. P(3 rock)

c. P(1 classical, 2 jazz)

Unit 6 Day 5 – Adding Probability Name ____________

Mutually Exclusive Events: 2 events that can't happen at the same time

Probability of Mutually Exclusive Events: *OR → key word

P(A or B) = P(A)+P(B)

1. Shannon has a stack of 8 baseball cards, 5 basketball cards, and 6 soccer cards. If she selects a card at random from the stack, what is the probability that is a baseball card or soccer card? 19 total

$P(\text{Base or Soccer}) = \frac{8}{19} + \frac{6}{19} = \frac{14}{19}$

2. Zoe has a standard deck of playing cards. If she selects a card at random, what is the probability she selects a card that is a heart or club?

$P(\heartsuit) + P(\text{club}) = \frac{13}{52} + \frac{13}{52} = \frac{1}{2}$

3. There are 7 girls and 6 boys on the junior class homecoming committee. A subcommittee of 4 people is being chosen at random to decide the theme for the class float. What is the probability that the subcommittee will have at least 2 girls?

GGBB, GBGB, GBBG, BBGG, BGBG, BGGB

2G2B + 3G1B + 4G

BGGG, GBGG, GGBG, GGGB

$6\left(\frac{7}{13}\cdot\frac{6}{12}\cdot\frac{6}{11}\cdot\frac{5}{10}\right) + 4\left(\frac{7}{13}\cdot\frac{6}{12}\cdot\frac{6}{11}\cdot\frac{5}{10}\right) + \left(\frac{7}{13}\cdot\frac{6}{12}\cdot\frac{5}{11}\cdot\frac{4}{10}\right) = \frac{112}{143}$

Mutually Inclusive Events: Events that can happen at the same time

Probability of Inclusive Events:

If A and B are inclusive then, P(A or B) = P(A)+P(B) − P(Both) overlap, &

4. What is the probability of drawing a queen or a diamond from a standard deck of cards?

$P(Q) + P(\diamondsuit) - P(\text{Both}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

5. What is the probability of drawing a red card or an ace?

$P(R) + P(\text{Ace}) - P(\text{Both}) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{7}{13}$

6. What is the probability of drawing a black card or a club?

$P(B) + P(\text{club}) - P(\text{Both}) = \frac{26}{52} + \frac{13}{52} - \frac{13}{52} = \frac{1}{2}$

7. The enrollment at Southern High is 1400 students. 550 take French, 700 take AFM, and 400 take both. What is the probability that a student selected at random takes French or AFM?

$P(F) + P(AFM) - P(Both) = \frac{550}{1400} + \frac{700}{1400} - \frac{400}{1400} = \frac{850}{1400} = \frac{17}{28}$

8. The probability that it will rain on Sat. is $\frac{2}{3}$. The probability that the temperature will reach 100°F is $\frac{4}{9}$. The probability that it will rain or reach 100°F is $\frac{4}{5}$. What is the probability that it will rain and reach 100°F on Saturday?

$P(R) + P(100°)a - P(Both) = P(R \text{ or } 100°)$

$\frac{6}{9} \cancel{\frac{2}{3}} + \frac{4}{9} - x = \frac{4}{5}$

$\frac{10}{9} - x = \frac{4}{5}$

$-x = \frac{-14}{45}$

$\boxed{x = 14/45}$

Two cards are drawn from a standard deck of 52 cards. What is the probability that each occurs?

9. 2 spades D

$\frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$

10. 2 spades or 2 red

$P(2\text{ Spades}) + P(2\text{ Reds})$ ME

$\frac{1}{17} + \frac{26}{52} \cdot \frac{25}{51} = \frac{31}{102}$

11. 2 red cards or 2 jacks MI

$P(2R) + P(2\text{ Jack}) - P(Both)$

$\frac{26}{52} \cdot \frac{25}{51} + \frac{4}{52} \cdot \frac{3}{51} - \frac{2}{52} \cdot \frac{1}{51} = \frac{55}{221}$

12. 2 spades or 2 face cards MI

$P(2S) + P(2\text{ Face}) - P(Both)$

$\frac{13}{52} \cdot \frac{12}{51} + \frac{12}{52} \cdot \frac{11}{51} - \frac{3}{52} \cdot \frac{2}{51} = \frac{47}{442}$

Adding Probabilities Practice

I. Lisa has 9 rings in her jewelry box. Five are gold and 4 are silver. If she randomly selects 3 rings to wear to a party, find each probability.

1. P (2 silver or 2 gold)

2. P (all gold or all silver)

3. P (at least 1 silver)

Unit 6 Day 6 – Expected Value & Fairness

Name ______________

Expected Value: the sum of the product of outcomes w/ their probabilities

1. The daily earnings of an employee who works on a commission basis are given by the following probability distribution. Find the employee's expected earnings. *All probabilities must add to one*

x (in $)	0	25	50	75	100	125	150
P(x)	0.07	.12	.17	.14	.28	.18	.04

$E(x) = 0(.07) + 25(.12) + 50(.17) + 75(.14) + 100(.28) + 125(.18) + 150(.04) = \$78.5$

2. Consider the experiment of rolling two dice and adding the numbers on top of the faces. Calculate the expected value of the probability distribution.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Expected Value:

$2(\frac{1}{36}) + 3(\frac{2}{36}) + 4(\frac{3}{36}) + 5(\frac{4}{36}) + 6(\frac{5}{36}) + 7(\frac{1}{6}) + 8(\frac{5}{36}) + 9(\frac{1}{9}) + 10(\frac{1}{12}) + 11(\frac{1}{18}) + 12(\frac{1}{36})$

$= \frac{252}{36} = \boxed{7}$

Sum	2	3	4	5	6	7	8	9	10	11	12
Prob	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

3. If the sum of two rolled dice is 8 or more, you win $2; if not, you lose $1. Find the expected value of the game for you.

Event	sum ≥ 8	Sum < 8
Payoff	2	-1
Probability	15/36	21/36

Expected Value:

$2(\frac{15}{36}) + -1(\frac{21}{36}) = \$.25$

4. Consider a family with 2 children. Assume that births of boys and girls are equally likely. Find the expected value for the number of girls.

BB GB
GG BG

# girls	0	1	2
Prob.	1/4	2/4	1/4

Expected value:

$0(\frac{1}{4}) + 1(\frac{2}{4}) + 2(\frac{1}{4})$

$= 1$ Girl

5. Consider a family with 3 children. Assume that births of boys and girls are equally likely. Find the expected value for the number of boys.

GGG GGB BBG GBB
GBG BGG BGB BBB

# boys	0	1	2	3
Prob.	1/8	3/8	3/8	1/8

Expected Value:

$0(\frac{1}{8}) + 1(\frac{3}{8}) + 2(\frac{3}{8}) + 3(\frac{1}{8})$

$= 1.5$ or $3/2$

In a lottery, the value of a ticket is a random variable, defined to be the amount of money you win less the cost of playing.

6. Suppose that in a lottery for charity with 225 tickets, each ticket costs $1. First prize is $50, second prize $30, and third prize is $20.

	Net gain 1st Prize	Net gain 2nd Prize	Net gain 3rd Prize	No prize
$	49	29	19	−1
Probability	1/225	1/225	1/225	222/225

Expected Value of one lottery ticket:

$= 49(\frac{1}{225}) + 29(\frac{1}{225}) + 19(\frac{1}{225}) - 1(\frac{222}{225}) = \$ -5/9$

7. In a lottery, 120 tickets are sold at $1 each. First prize is $50 and second is $20. Find the expected value of a ticket.

	1st Place	2nd place	no prize
$	49	19	−1
Prob	1/120	1/120	118/120

Expected Value of one lottery ticket:

$= 49(\frac{1}{120}) + 19(\frac{1}{120}) - 1(\frac{118}{120})$

$= -\frac{5}{12}$

8. The Island Club is holding a fund-raising raffle. Ten thousand tickets have been sold for $2 each. There will be a first prize of $3000, 3 second prizes of $1000 each, 5 third prizes of $500 each, and 20 consolation prizes of $100 each.. Find the expected value of the raffle ticket.

	1st	2nd	3rd	consulation	loser
Net$	2998	998	498	98	-2
prob	1/10,000	3/10,000	5/10,000	20/10,000	9971/10,000

$= \$ -.95$

Expected Value:

$$2998\left(\frac{1}{10{,}000}\right) + 998\left(\frac{3}{10{,}000}\right) + 498\left(\frac{5}{10{,}000}\right) + 98\left(\frac{20}{10000}\right) - 2\left(\frac{9971}{10000}\right)$$

9. In a certain state's lottery, six numbers are randomly chosen without repetition from the numbers 1 to 40. If you correctly pick all 6 numbers, only 5 of the 6, or only 4 of the 6, then you will $1 million, $1000 or $100, respectively. What is the expected value of a $1 lottery ticket?

	6	5	4	none
$	999,999	999	99	-1
prob				

all 6 $\frac{{}_6C_6}{{}_{40}C_6} = \frac{1}{3{,}838{,}380}$

5 $\frac{{}_6C_5 \cdot {}_{34}C_1}{{}_{40}C_6} = \frac{204}{3838380}$

4 $\frac{{}_6C_4 \cdot {}_{34}C_2}{{}_{40}C_6} = \frac{8415}{3838380}$

Expected Value:

$ -.47

Fairness in a Game: For a game to be fair, no one has advantage of another player. This translates to having an expected value of __0__. So to determine if something is fair, calculate the expected value.

11. If the sum of two rolled dice is 8 or more, you win $2; if not, you lost $1.

a. Show that this is not a fair game.

	Sum ≥8	<8
$	2	-1
Prob	15/36	21/36

$EV = 2\left(\frac{15}{36}\right) + -1\left(\frac{21}{36}\right)$

$EV = \frac{1}{4}$

not fair since $EV \neq 0$

b. To have a fair game, the $2 winnings should instead be what amount?

$X\left(\frac{15}{36}\right) - \frac{21}{36} = 0$

$\frac{15X}{\cancel{36}} = \frac{21}{\cancel{36}}$

$15x = 21$

$\boxed{X = 7/5 \text{ or } 1.4}$

12. Two coins are tossed. If both land heads up, then player A wins $4 from player B. If exactly one coin lands heads up, then B wins $1 from A. If both land tails up, then B wins $2 from A. Is this a fair game?

For Player A

	2H	1H	No H
$	4	-1	-2
Prob	1/4	2/4	1/4

HH
HT
TH
TT

yes!

$EV = 4\left(\frac{1}{4}\right) - 1\left(\frac{2}{4}\right) - 2\left(\frac{1}{4}\right) = 0$

13. Two dice are rolled. If the sum of the numbers showing on the dice is odd, player A wins $1 from player B. If both dice show the same number, A wins $3 from B. Otherwise B wins $3 from A. Is this a fair game?

For A

	sum odd	same #	other
$	1	3	-3
Prob	18/36	6/36	12/36

$EV: 1\left(\frac{18}{36}\right) + 3\left(\frac{6}{36}\right) - 3\left(\frac{12}{36}\right)$

EV = $0

yes, its fair!

14. Mike and Bill play a card game with a standard deck of 52 cards. Mike selects a card from a well-shuffled deck and receives "A" dollars from Bill if the card selected is a diamond; otherwise, Mike pays Bill a dollar. Determine the value of A if the game is to be fair.

EV=0

	◇	other
$	A	-1
Prob	1/4	3/4

$A\left(\frac{1}{4}\right) - 1\left(\frac{3}{4}\right) = 0$

$\frac{A}{4} - \frac{3}{4} = 0$

$\frac{A}{4} = \frac{3}{4}$

$\boxed{A = 3}$

15. Vivian and Katie play a dice game with two dice. Vivian rolls the die and will receive $0.12 each time the sum of 3,4 or 5 occur, but will lose "B" when any other sum shows. Determine the value of B if the game is to be fair.

EV=0

	3,4,5 sum	other sum
$	.12	-B
Prob	9/36	27/36

$0 = .12\left(\frac{9}{36}\right) - B\left(\frac{27}{36}\right)$

$0 = .03 - \frac{27B}{36}$

$36 - .03 = \frac{-27B}{36} \cdot 36$

$\frac{-.27}{.25} = -27B$

$\frac{1}{25} = B$

$.04 = B$

Unit 6 Day 7 – Geometric Probability

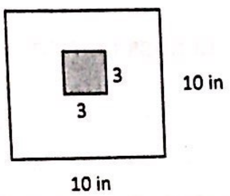
Name ______________________

To Find Geometric Probability: Find the area of success then divide it by the total area.

$$Probability = \frac{area\ of\ success}{total\ area}$$

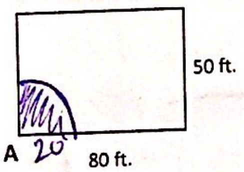
*Area of a rectangle: $l \cdot w$ *Area of a circle: πr^2

1. Sue is throwing darts at a square game board as shown in the figure. What is the chance that she will throw a dart in the shaded area in the center of the board? (Assume that every dart thrown lands somewhere on the board.)



$$\frac{A_S}{A_T} = \frac{9}{100}$$

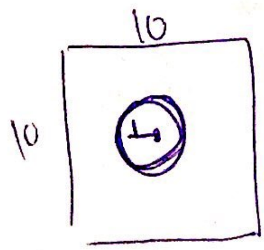
2. Moe Mentum owns a goat that will eat anything especially the tennis balls that are hit into Moe's yard from a neighboring tennis court. To keep his goat from eating a lot of tennis balls, Moe decides to tether it in a corner of a yard at Point A as shown. The tether is 20 feet long. What is the probability that a tennis ball hit into Moe's yard will be within reach of the tethered goat?



$$\frac{A_S}{A_T} = \frac{100\pi}{4000} = \frac{\pi}{40} \approx .0785$$

$$\frac{\pi(20)^2}{4}$$

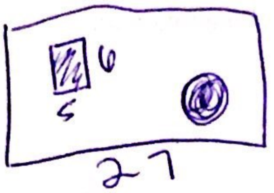
3. Washington, D.C. was originally laid out as a square with sides ten miles in length. On a visit to Washington, Myles Away plans to visit all the important sites such as the White House, the Smithsonian Museum, the Capitol, and the National Zoo. What is the probability that any one of these sites in within a mile of the center of Washington?



$$\frac{A_{circle}}{A_{total}} = \frac{\pi}{100} \approx .0314$$

Scanned with CamScanner

4. A rectangular field measures 27 feet by 15 feet. A small shed is on the field. Its dimensions are 6 feet by 5 feet. There is also an oak tree in the field whose branches form a circular canopy with a diameter of 10 feet. (Assume the shed is not under the tree.)



$A_{shed} = 30$

$A_{tree} = \pi(5)^2 = 25\pi$

$A_{total} = 27 \cdot 15 = 405$

a. What is the probability that a single drop of rain that lands in the field would hit the shed?

$\frac{30}{405} = \frac{2}{27}$

b. What is the probability that a single drop of rain that lands in the field would *not* hit the shed?

$1 - \frac{2}{27} = \frac{25}{27}$

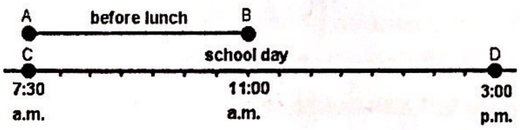
c. What is the probability that a single drop of rain that lands in the field would miss both the shed and the tree?

$1 - \frac{2}{27} - \frac{25\pi}{405} = .732$

Some problems involve linear information. The best way to approach these types of problems is to draw the picture with the given information then use:

$$\textbf{Probability} = \frac{\textbf{length of line that represents success}}{\textbf{total length}}$$

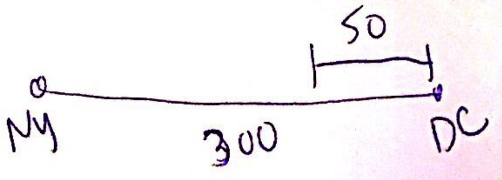
5. Suppose that your school day begins at 7:30 a.m. and ends at 3:00 p.m. You eat lunch at 11:00 A.M. If there is a fire drill at a random time during the day, what is the probability that it begins before lunch? You can use line segments to model the probability.



$\frac{\text{Before}}{\text{total day}} = \frac{3.5}{7.5} = \frac{7}{15}$

6. Holly Mackerel and Patty Cake are driving from New York City to Washington, D.C., a distance of about 300 miles. Their car has a broken gas gauge, but Holly knows her car's gas tank holds exactly enough gas to make the trip without having to stop for gas. Unfortunately, they hit bad weather, which causes traffic delays, and they run out of gas. What is the probability that they will be within 50 miles of Washington when they run out of gas?

$\frac{50}{300} = \frac{5}{30} = \frac{1}{6}$



Unit 6 Day 8 – Binomial Probability

Name ____________

A binomial experiment exists if and only if these conditions occur:

- The experiment consists of n identical trials.
- Each trial results in one of two possible outcomes: success or failure.
- The trials are **independent**. (probabilities must remain **constant**)

Tell whether the following could be a binomial experiment.

H or T

B G

yes 1. Flipping a coin

yes 2. Probability of having a girl in a family of 4.

NO 3. Probability of pulling 2 diamonds from a deck of cards **without replacement**.

dependent

yes 4. Probability of pulling 2 diamonds from a deck of cards **with replacement**.

The following formula can be used to find the probability of success in a binomial experiment.

$$P(r) = {}_nC_r\, p^r (1-p)^{n-r}$$

n = # trials   r = # successes   p = probability of success

(1 – p) = probability of failure   P(r) = probability of r successes

5. If there are 10 true-false questions on a quiz, what is the probability that exactly 8 answers are correct?

$n=10$
$r=8$
$p=\frac{1}{2}$
$1-p=\frac{1}{2}$

$$P(r) = {}_{10}C_8\left(\tfrac{1}{2}\right)^8\left(\tfrac{1}{2}\right)^2$$

correct wrong

$$= \frac{10!}{2!8!}\left(\tfrac{1}{2}\right)^8\left(\tfrac{1}{2}\right)^2 = 45\left(\tfrac{1}{256}\right)\left(\tfrac{1}{4}\right)$$

$$= \frac{45}{1024} = .044$$

6. While pitching for the Toronto Blue Jays, 4 of every 7 pitches Juan Guzman threw in the first 5 innings were strikes. What is the probability that in the next inning, Juan will throw a strike and 4 balls in his next 5 pitches?

success = strike

$n = 5$ pitches
$r=1$
$p=\frac{4}{7}$
$1-p=3/7$

$$P(\text{strike}, 4\text{ balls}) = {}_5C_1\left(\tfrac{4}{7}\right)^1\left(\tfrac{3}{7}\right)^4$$

$$= \frac{5!}{4!1!}\left(\tfrac{4}{7}\right)\left(\tfrac{3}{7}\right)^4$$

$$= .0964$$

TO DO **THIS WITH A GRAPHING CALC: 2^{nd} →VARS → Distr → Binompdf (type A)**
Binompdf **(n, p, r) to find the probability of getting r successes**
x → calc calls ~~x~~ that x

7. If 6 coins are tossed, what is the probability of each?

a. 3 heads and 3 tails success = H
$n = 6$
$r = 3$
$p = \frac{1}{2}$
$1-p = \frac{1}{2}$
$.3125 = \frac{5}{16}$

b. at least 4 heads 4H, 5H, 6H
$n = 6$
$r = 4, 5, 6$ ↑ add!
$p = \frac{1}{2}$
$\frac{11}{32} = .34375$

c. no more than 4 heads
$n = 6$
$r = 0, 1, 2, 3, 4$
$p = \frac{1}{2}$ ↑ add!
$\frac{57}{64} = .890625$

d. all heads or all tails
$n = 6$
$r = 0, 6$
$p = \frac{1}{2}$
$\frac{1}{32} = .03125$

8. The probability if Chris making a free throw is $\frac{2}{3}$. If she shoots five times, what is the probability of each?

a. all missed success = make FT
$n = 5$ shots
$r = 0$
$p = \frac{2}{3}$
$\frac{1}{243} = .0041$

b. all made
$n = 5$
$r = 5$
$p = 2/3$
$\frac{32}{243} = .13169$

c. exactly 4 made
$n = 5$
$r = 4$
$p = 2/3$
$\frac{80}{243} = .3292$

d. at least 2 made
$n = 5$
$r = 2, 3, 4, 5$ ↑ add!
$p = 2/3$
$\frac{232}{243} = .9547$

9. If you have 6 blue marbles, 4 red marbles, and 5 yellow marbles. Tell whether each situation is binomial and if so find the probability.

a. P(2 blue); with replacement
yes
$n = 2$
$r = 2$
$p = \frac{6}{15}$
$\frac{4}{25}$

b. P(2 red); without replacement
no, dependent

c. P(1 red, 1 yellow); with rep
~~yes~~ no
2 diff. successes