AFM Unit 5 Day 1 – Arithmetic Sequences

Name Key

Sequence: A sequence is a set of numbers in a certain order. Each number in a sequence is called a term.

What kinds of sequences do you remember? Arithmetic (adding) & Geometric (Multiplying)

1. Find the next three terms in each sequence.

a. 80, 77, 74, 71, 68, … (-3)
65, 62, 59
(−3)

b. 4, 8, 16, 32, 64, … (x2)
128, 256, 512

c. 0, 3, 7, 12, 18, … (+3 +4 +5 +6 +7)
25, 33, 42

d. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$
$\frac{1}{64}, \frac{1}{128}, \frac{1}{256}$

Recursive Formula: **MUST STATE FIRST TERM AND FORMULA**

* a_n - nth term
* a_{n-1} - previous term
* a_1 - first term
* n - term #

The term you are given, a_1, is considered NOW aka a_{n-1} the next consecutive term, a_2, aka a_n is called NEXT

Ex: $a_n = a_{n-1} + 3$ can be interpreted as NEXT = NOW + 3

2. If $a_1 = 22$ and $a_n = a_{n-1} - 3$, find the next three terms in the sequence.

22, 19, 16, 13

3. If $a_1 = 3$ and $a_n = a_{n-1} + 8$, find the next five terms in the sequence.

3, 11, 19, 27, 35, 43

4. If $a_0 = 2$ and $a_n = n^2 + 2n - a_{n-1}$, find the next five terms. 2, 1, 7, 8, 16, 19

$a_1 = 1^2 + 2(1) - 2$
$a_1 = 1 + 2 - 2 = 1$
$a_2 = 2^2 + 2(2) - 1 = 7$
$a_3 = 3^2 + 2(3) - 7$
$a_3 = 8$
$a_4 = 4^2 + 2(4) - 8 = 16$
$a_5 = 5^2 + 2(5) - 16$
$= 19$

5. If $a_1 = -2$ and $a_n = -2a_{n-1} + 3n$, find the next four terms in the sequence.

−2, 10, −11, 34, −53

$a_2 = -2(-2) + 3(2) = 10$
$a_3 = -2(10) + 3(3) = -11$
$a_4 = -2(-11) + 3(4) = 34$
$a_5 = -2(34) + 3(5) = -53$

Explicit Formula - Gives the rule in terms of the nth term (IN TERMS OF N)

6. If $a_n = 16 - 3n$, find the first four terms. plug in!

13, 10, 7, 4.

7. If the domain values are $\{-1,0,3,5\}$, find the corresponding range values for $a_n = 2n - 5$.

plug in! -7, -5, 1, 5

Arithmetic sequence: a sequence where you add/subtract a common diff. each time

Find the common difference, then find the next three terms:

8. a. $-12, \ -18, \ -24, \ . \ . \ .$
$d = -6$
-30, -36, -42

b. 7, 10, 13, ...
$d = 3$
16, 19, 22

c. $r-4, \ r-2, \ r, \ . \ . \ .$
$d = +2$
$r+2, r+4, r+6$

Recursive rule:
$a_n = a_{n-1} + d$
$a_1 = \#$

Explicit rule:
$a_n =$ then simplify
$\rightarrow a_0 + dn$

The nth term of an Arithmetic Sequence: $a_n = a_1 + (n-1)d$

Write the recursive and explicit rule for the following sequences. Then find the 30th term.

+3
9. 14, 17, 20, 23, ...
11
$a_n = a_{n-1} + 3$
$a_1 = 14$ $a_{30} = 101$
$a_n = 3n + 11$

-7
10. 2, −5, −12, −19, ...
$a_n = a_{n-1} - 7$ $a_{30} = -201$
$a_1 = 2$
$a_n = -7n + 9$

11. Find the 41st term in the sequence: 11, 4, −3, ...
-7
18,
$a_n = -7n + 18$

$a_{41} = -269$

12. Find the 24th term in the sequence for which $a_1 = -27$ and $d = 3$.

$a_{24} = -27 + (24-1)3$

$= -27 + 23(3)$

$a_{24} = 42$

13. Find the first term in the sequence for which $a_{44} = 229$ and $d = 8$.

$229 = a_1 + (44-1)8$

$229 = a_1 + 43 \cdot 8$

$a_1 = -115$

Arithmetic mean: terms between any nonconsecutive terms of an arith. sequence

The terms between any two nonconsecutive terms of an arithmetic sequence are called **arithmetic means.**

14. Form an arithmetic sequence that has five arithmetic means between -11 & 19.

$d = 19 - (-11) = \frac{30}{6}$

$d = 5$

$\underset{a_1}{\underline{-11}}$, $\underline{-6}$, $\underline{-1}$, $\underline{4}$, $\underline{9}$, $\underline{14}$, $\underset{a_7}{\underline{19}}$

$19 = -11 + (7-1)d$

$30 = 6d$

$5 = d$

15. Form an arithmetic sequence that has six arithmetic means between -12 & 23.

$\underset{a_1}{\underline{-12}}$, $\underline{-7}$, $\underline{-2}$, $\underline{3}$, $\underline{8}$, $\underline{13}$, $\underline{18}$, $\underset{a_8}{\underline{23}}$

$23 = -12 + (8-1)d$

$35 = 7d$

$5 = d$

AFM Unit 5 Day 2 – Geometric Sequences

Name________________

Geometric Sequence: a pattern that multiplies by the same # each time

Common Ratio: the # multiplied btw each term

***can be found by dividing any term by the** previous **term**

1. Find the common ratio, then find the next three terms: 27, 135, 675, . 3375, 16875, 84375

$\frac{135}{27} = 5$ $\frac{675}{135} = 5$ $r = 5$

2. Find the common ratio, then find the next three terms: 162, 54, 18, … . 6, 2, $\frac{2}{3}$

$\frac{54}{162} = \frac{1}{3}$ $\frac{18}{54} = \frac{1}{3}$ $r = \frac{1}{3}$

Recursive Rule:

$a_n = a_{n-1} \cdot r$

$a_1 = 1^{st}$

Explicit Rule:

$a_1 = 1^{st}$ term $n = \#$ term

$a_n =$ nth term

$r =$ common ratio

The nth term in a Geometric Sequence: $a_n = a_1 r^{n-1}$

Write the recursive and explicit rule for the following sequences. Then find the 10^{th} term.

3. $-4, 8, -16, 32, \ldots$

R: $a_n = a_{n-1}(-2)$

$a_1 = -4$

E: $a_n = -4(-2)^{n-1}$

$a_{10} = 2048$

4. $1000, 800, 640, 512, \ldots$

R: $a_n = .8a_{n-1}$

$a_1 = 1000$

E: $a_n = 1000(.8)^{n-1}$

$a_{10} = 134.22$

5. $\frac{1}{2}, 1, 2, 4, \ldots$ $r = 2$

R: $a_n = 2a_{n-1}$

$a_1 = \frac{1}{2}$

E: $a_n = \frac{1}{2}(2)^{n-1}$

$a_{10} = 256$

6. The first term of a geometric sequence is 12 and the common ratio is $-\frac{3}{2}$. Find the next four terms.

12, −18, 27, −40.5, 60.75

7. Find the 14^{th} term in the geometric sequence 1, $\frac{1}{3}$, $\frac{1}{9}$, ...

$r = \frac{1}{3}$ $a_{14} = 1\left(\frac{1}{3}\right)^{14-1}$ $\frac{1}{1594323}$ or $a_{14} = .000000627$

8. Find the 12^{th} term in the geometric sequence 24, 12, 6, … .

$r = \frac{1}{2}$ $a_{12} = 24\left(\frac{1}{2}\right)^{12-1}$

$a_{12} = \frac{3}{256} \approx .01172$

Find t
6)

9. Determine the recursive and explicit equation given $a_6 = 15552$ *and* $a_1 = 2$.

$\frac{15552}{2} = \frac{2(r)^5}{2}$

$\sqrt[5]{7776} = r^5$

$r = 6$

R: $a_n = 6a_{n-1}$
$a_1 = 2$

E: $a_n = 2(6)^{n-1}$

10. Determine the recursive and explicit equation given $a_6 = 3072$ *and* $a_1 = 3$.

$\frac{3072}{3} = 3r^5$

$\sqrt[5]{1024} = r^5$

$r = 4$

R: $a_n = 4a_{n-1}$
$a_1 = 3$

E: $a_n = 3(4)^{n-1}$

Geometric Mean: terms btw any nonconsecutive terms of a geo seq.

11. Form a sequence that has two geometric means between 136 & 459 .

$\underset{a_1}{136}$, 204, 306, $\underset{a_4}{459}$

$459 = 136r^{4-1}$

$3.375 = r^3$

$r = 1.5$

12. Form a sequence that has two geometric means between $\frac{1}{2}$ *and* $-\frac{125}{16}$.

$\underset{a_1}{\frac{1}{2}}$, -1.25, 3.125, $\underset{a_4}{-\frac{125}{16}}$

$-\frac{125}{16} = \frac{1}{2}r^{4-1}$

$-15.625 = r^3$

$r = -2.5$

Classwork/Homework Day 2

1. Determine whether $\sqrt{2}, 2, \sqrt{8}, \ldots$ forms a geometric sequence.

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AFM Unit 5 Day 3 – Arithmetic Series Name____________

Arithmetic Series: The indicated sum of the terms of an arithmetic sequence.

Arithmetic sequence: 3,7,11,15,19

Arithmetic series: 3 + 7 + 11 + 15 + 19

Infinite Series: Doesn't end

Finite Series: Ends

1. Find the sum of the set of consecutive integers from 1 to 10.

$1+2+3+4+5+6+7+8+9+10=55$

The sum of the first *n* terms of an arithmetic series is given by the formula: $S_n = \frac{n}{2}(a_1 + a_n)$

n = # terms

S_n = Sum of n terms

a_1 = 1st term

a_n = last term #

2. Find the sum of the set of consecutive integers from 1 to 100.

$S_{100} = \frac{100}{2}(1+100)$

$= 50(101) = 5050$

3. Find the sum of the first 27 terms in the series $-14-8-2-...+142$.

$S_{27} = \frac{27}{2}(-14+142) = 1728$

4. Find the sum of the first 32 terms in the series $-12-6-0-...$ $d=6$

$a_n = -12+(32-1)(6)$

$a_{32} = 174$

$S_{32} = \frac{32}{2}(-12+174)$

$= 2592$

5. Find the sum of the series $-2 + 2.5 + 7 + 11.5 + \cdots + 268$. $d=4.5$

$268 = -2+(n-1)(4.5)$

$\frac{270}{4.5} = 4.5(n-1)$

$60 = n-1$

$n = 61$

$S = \frac{61}{2}(-2+268)$

$= 8113$

6. $S_n = -1207,\ a_1 = 14,\ d = -3$. Find n.

$a_n = 14 + (n-1)(-3)$

$a_n = 17 - 3n$

$-1207 = \frac{n}{2}(14 + a_n)$

$-1207 = \frac{n}{2}(14 + 17 - 3n)$

$-2414 = n(-3n + 31)$

$0 = -3n^2 + 31n + 2414$

$n = \frac{-31 \pm \sqrt{31^2 - 4(-3)(2414)}}{2(-3)}$

$\boxed{n = 34}$

7. Find the first three terms of the arithmetic series given $n = 16, a_n = 15, S_n = -20$.

$-20 = \frac{16}{2}(a_1 + 15)$

$-2.5 = a_1 + 15$

$-17.5 = a_1$

$a_{16} = -17.5 + (15)d$

$15 \quad \frac{32.5}{15} = 15d$

$d = 13/6$

$-17.5,\ -15.33,\ -13.167$

8. Given $S_n = 822$, $n = 12$, and $a_1 = 8$, find a_n.

$822 = \frac{12}{2}(8 + a_n)$

$137 = 8 + a_n$

$\boxed{129 = a_n}$

9. Given $a_1 = 12$ and $a_n = 86$, and $S_n = 931$, find n.

$931 = \frac{n}{2}(12 + 86)$

$1862 = 98n$

$\boxed{19 = n}$

10. A pile of logs has 1 log in the top layer, 2 in the second layer, 3 in the third and so on. How many logs are in the pile if it contains 20 layers?

$a_{20} = 1 + (20-1)(1)$

$a_{20} = 1 + 19 = 20$

$S_{20} = \frac{20}{2}(1 + 20)$

$= 210$ logs

11. A farmer gathers 35 bushels of sweet potatoes on the first day of harvest. On each successive day, the amount to be gathered will be 4 bushels more than the preceding day. If the harvest lasts 12 day, what is the total number of bushels he can expect to collect?

$a_n = 35 + (12-1)(4)$

$a_{12} = 79$

$S_{12} = \frac{12}{2}(35 + 79)$

$S_{12} = 684$ bushels

13. Nicole starts a college savings account for her daughter on her sixth birthday. She plans to deposit \$25 the first month and then increase the deposit by \$5 each month. How much will she have deposited in twelve years?

144 month

$a_{144} = 25 + (14$

$= 740$

$S_{144} = \frac{144}{2}(25 + 740)$

$= \$55,080$

1C

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AFM Unit 5 Day 4 – Geometric Series

Name________________

Geometric Series: The indicated sum of the terms of a geometric sequence.

Geometric sequence: 3, 6, 12, 24

Arithmetic series: 3 + 6 + 12 + 24

The sum of the first n terms of a geometric series: $S_n = \frac{a_1(1-r^n)}{1-r},\ r \neq 1$

$n =$ # terms

$a_1 =$ 1st term

$S_n =$ sum of n terms

$r =$ common ratio

1. Find the sum of the first 8 terms of the series $3-6+12-\ . . . .$

$\times -2$

$$S_8 = \frac{3(1-(-2)^8)}{1-(-2)} = -255$$

2. Find the sum of the first six terms of the series $-\frac{3}{4}-\frac{9}{20}-\frac{27}{100}-\ \cdots$

$\times 3/5$

$$S_6 = \frac{-\frac{3}{4}(1-\frac{3}{5}^6)}{1-3/5} = -1.78752$$

3. Find the sum of the series given $a_1 = 60, n = 6, and\ r = 1/2.$

$$S = \frac{60(1-\frac{1}{2}^6)}{1-\frac{1}{2}} = 118.125$$

4. Determine the number of terms in the series given: -2 - 4 - 8 - 16 - ...., S_n = -254

$\times 2$

$\cancel{6}(-254) = \frac{-2(1-2^n)}{\cancel{-3}\ \cancel{1-2}}$

$\frac{-254}{-2} = \frac{-2(1-4^n)}{-2}$

$-127 = 1-2^n$

$-128 = -2^n$

$\log_2(128) = n$

$\boxed{n=7}$

5. Determine the number of terms in the series given: $a_1 = 2$, r = 4, Sn = 2730

$-3(2730) = \frac{2(1-4^n)}{-3\ \cancel{1-4}}$

$\frac{-8190}{2} = \frac{\cancel{2}(1-4^n)}{\cancel{2}}$

$-4095 = 1-4^n$

$-4096 = -4^n$

$4096 = 4^n$

$\log_4(4096) = n$

$\boxed{n=6}$

13

In some cases, we can evaluate the sum of an infinite geometric series!

Example: the sequence $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots \frac{1}{n}$

*What happens as "n" gets larger? gets closer to zero!

*As a sequence approaches a number, we say that the sequence as a limit.

$\lim_{n\to\infty} \frac{1}{n} = 0$ is read: *"the limit of 1 over n, as n approaches infinity equals zero"*

Rules for Finding Limits (same as asymptote rules): HA Rules

a. If the largest exponents are the same in the numerator and denominator, the limit is the ratio of the coefficients of the terms containing the largest exponent.

b. If largest exponent is in the numerator, there is no limit.

c. If the largest exponent is in the denominator, the limit is zero.

Examples: Find the following limits:

a. $\lim_{n\to\infty} \frac{1}{4^n}$

0

b. $\lim_{n\to\infty} \frac{3n}{4n^2+1}$

0

c. $\lim_{n\to\infty} \frac{4n}{3n-2}$

4/3

d. $\lim_{n\to\infty} \frac{n^2}{5n}$

no limit

1. Find each limit.

a. $\lim_{n\to\infty} \frac{1-2n}{5n}$

-2/5

b. $\lim_{n\to\infty} \frac{4n^2-6}{3n}$

no limit

c. $\lim_{n\to\infty} \frac{n^2-3n+4}{n^2}$

1

d. $\lim_{n\to\infty} \frac{n^2+4}{n^3}$

zero

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*If a geometric series is approaching a limit, then we can find the sum of the infinite series.

When $|r| < 1$, the series converges, or gets closer and closer to the sum.
When $|r| > 1$, the series diverges, or approaches no limit.

To find the sum of an infinite geometric series, where $r \neq 1$, $S_\infty = \dfrac{a_1}{1-r}$ *

1. Determine whether each series converges or diverges. Then find the sum if it exists.

a. $\frac{1}{20}+\frac{1}{40}+\frac{1}{80}+\ ...$

$r=\frac{1}{2}$ converges

$S=\dfrac{\frac{1}{20}}{1-\frac{1}{2}}=\frac{1}{10}$

b. $2\sqrt{2}+8+16\sqrt{2}\ ...$

$r=2\sqrt{2}$

diverges

c. $\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\ ...$

$r=\frac{1}{2}$ converges

$S=\dfrac{2/3}{1-\frac{1}{2}}=\frac{4}{3}$

d. $1+3+9+27+\ ...$

x3 x3

$r=3$ diverges

e. 1 - 1/3 + 1/9 - ...

$r=-\frac{1}{3}$ converges

$S=\dfrac{1}{1-(-\frac{1}{3})}=3/4$

f. 1 + 1/5 + 1/25 + ...

$r=\frac{1}{5}$ converges

$S=\dfrac{1}{1-\frac{1}{5}}=5/4$

g. 4 + 8 + 16 + ...

x2 diverges

AFM Homework – Geometric Series

Find the sum of the geometric series described. Show all work.

1. $a_1 = 3$, $r = 2$, $n = 6$

2. $a_1 = 8$, $r = 1.5$, $a_n = 40.5$

3. $1000+800+640+\cdots$ *for 12 terms.*

4. $\frac{1}{2}+2+8+16\ ...$ *for 8 terms.*

15

AFM Unit 5 Day 5 – Sigma Notation Name____________________

Sigma Notation – also known as Summation Notation

*Simplifies the process of writing out the sum of a series

$$\sum_{n=1}^{10} 2n$$

10 → last value of n

$\sum_{n=1}^{10} 2n$ is read as the sum of $2n$ as n increases from 1 to 10

$n=1$ → 1st value of n

$2n$ → explicit equation

Write each in expanded form and then find the sum.

1. $\sum_{k=1}^{5} 2k$

$2+4+6+8+10=30$

$S=\frac{5}{2}(2+10)$

2. $\sum_{k=-1}^{2} (k+2)$

$1+2+3+4=10$

$S=\frac{4}{2}(1+4)$

3. $\sum_{k=0}^{20} (3k+2)$

$S=\frac{21}{2}(2+62)$

$=672$

4. $\sum_{k=2}^{5} 2^k$

$4+8+16+32=60$

$S=\frac{4(1-2^4)}{1-2}$

5. $\sum_{n=0}^{6} (4n^2+1)$ neither, must write out!

$1+5+17+37+65+101+145=371$

6. Express the series 5 + 7 + 9 + 11 + 13 using sigma notation.

$$\sum_{n=1}^{5} (2n+3)$$

$a_n=5+(n-1)2$

$a_n=5+2n-2$

$a_n=2n+3$

7. Express the series -4 – 7 – 10 – 13 – 16 – 19 using sigma notation.

$$\sum_{n=1}^{6}(-1-3n)$$

$a_n = -4+(n-1)(-3)$

$a_n = -4-3n+3$

$a_n = -1-3n$

8. Express the series 1 – 2 + 4 – ⋯ + 1024 using sigma notation.

$$\sum_{n=1}^{11}(-2)^{n-1}$$

$a_n = 1(-2)^{n-1}$

*Guess + check to determine # terms

Practice - Write each of the following in expanded form and find the sum.

9. $\sum_{n=4}^{7}(3^n+1)$

$82+244+730+2188$

$=3244$

10. $\sum_{n=1}^{\infty}2\left(\frac{1}{3}\right)^n$

$r=\frac{1}{3}$ $a_1=\frac{2}{3}$

$S=\frac{\frac{2}{3}}{1-\frac{1}{3}}=1$

Express the following using sigma notation.

10. $16+19+22+25+28$

$$\sum_{n=1}^{5}(3n+13)$$

11. $4-2+1-\ldots-\frac{1}{8}$

$$\sum_{n=1}^{6}4\left(-\frac{1}{2}\right)^{n-1}$$

12. $\frac{1}{4}+\frac{1}{2}+1,+\cdots+32$

$$\sum_{n=1}^{8}\frac{1}{4}(2)^{n-1}$$

13. $48-24+12-6+\cdots$

$$\sum_{n=1}^{\infty}48\left(-\frac{1}{2}\right)^{n-1}$$

Write each expression in expanded form and find the sum.

14. $\sum_{r=1}^{3}(r-3)$

$-2 - 1 + 0 = -3$

15. $\sum_{b=4}^{49}(4-2b)$

$S=\frac{46}{2}(-4-94)$

$=-2254$

16. $\sum_{b=2}^{5}(b^2+b)$

$6+12+20+30$

$=68$

17. $\sum_{n=3}^{6}(3^n+1)$

$28+82+244+730$

$=1084$

18. $\sum_{p=1}^{4}\left(3^{p-1}+\frac{1}{2}\right)$

$1.5+3.5+9.5+27.5$

$=42$

19. $\sum_{k=1}^{\infty}4\left(\frac{1}{2}\right)^k$

$S=\frac{2}{1-\frac{1}{2}}=4$

18

AFM Unit 5 Day 6 – Applications

Name___________________________

*Determine if the application is arithmetic or geometric.

*Does it want a total sum or particular term number?

*If you get stuck, write out the first few terms and try to find the pattern.

1. A Greek theater has 30 seats in the first row of the center section. Each row behind the first row gains two additional seats. How many seats are in the 12th row? How many total seats are there in the first 12 rows?

A
$d=2$

$a_{12} = 30+(12-1)(2)$

$a_{12} = 52$ seats

$S_n = \frac{12}{2}(30+52)$

$= 492$ seats

initial pop

2. A small town has a population of 2,000 people and is growing at a rate of 3% per year.
   a. Write an equation to model the town's growth.

   $100\% + 3\% = 1.03$

   $y = 2000(1.03)^n$

   b. What will the expected population be in 8 years?

   ~2533 people

3. A piece of real estate bought 5 years ago for $25,600 increased in value 25% each year since then. What is it worth now?

$a_n = 25600(1.25)^n$ plug ins

year 0

$y = \$78,125$

4. A car traveled 32 meters in the first second after the brakes were applied and in each second after that traveled half as far as it had in the second before. How far did the car travel in the ten seconds after the brakes were applied?

32, 16, 8, 4, ... Sum

$r = \frac{1}{2}$

$S_n = \frac{32(1-\frac{1}{2}^{10})}{1-\frac{1}{2}} = 63.94$ ft traveled

20

5. How many rows are in the corner section of a stadium containing 2040 seats if the first row has 10 seats and each successive row has 4 additional seats?

total

$a_n = 10 + (n-1)(4)$

$a_n = 6 + 4n$

$2040 = \frac{n}{2}(10 + 6 + 4n)$

$4080 = n(4n + 16)$

$0 = 4n^2 + 16n - 4080$

$n = \frac{-16 \pm \sqrt{16^2 - 4(4)(-4080)}}{2(4)}$

$\boxed{n = 30}$

6. You just received a job offer with a starting salary of $35,000 per year with a guaranteed raise of $1400 per year. How many years will it take before your salary $49,000?

A

$a_n = 35000 + (n-1)(1400)$

$49000 = 35000 + (n-1)(1400)$

$14000 = 1400(n-1)$

$10 = n - 1$

$\boxed{11 = n \text{ years}}$

7. Initially, a pendulum swings through an arc of 2 feet. On each successive swing, the length of the arc is .9 of the previous length.

$a_n = 2(.9)^{n-1}$

swing

a. What is the arc length after 10 swings?

.775 ft

b. On which swing is the arc length first less than 1 foot?

~~8~~ 9th swing

c. After 15 swings, what total length will the pendulum have swung?

sum

$S_{15} = \frac{2(1 - .9^{15})}{1 - .9} = 15.882 \text{ ft}$

d. When it stops, what total length will the pendulum have swung?

$S = \frac{2}{1 - .9} = 20 \text{ ft}$

8. A rubber ball is shot vertically to a height of 20 ft and allowed to drop. Each bounce is 80% as high as the previous bounce. What is the total vertical distance the ball travels?

$r = .8$

sum

G

$S = \frac{20}{1 - .8} = 100 \text{ ft}$

9. Geologists estimate that the continents of Europe and North America are drifting apart at a rate of an average of 12 miles every 1 million years, or about 0.75 inch per year. If the continents continue to drift apart at that rate, how many inches will they drift in 50 years?

A

$S = 50(.75) = 37.5$ inches total

10. Contagio... and that...

2

AFM Unit 5 Day 7 – Binomial Theorem

Name________________________________

Expand the following.

1. $(a+b)^0$

1

2. $(a+b)^1$

$a+b$

3. $(a+b)^2$

$a^2+2ab+b^2$

4. $(a+b)^3$

$(a^2+2ab+b^2)(a+b)$

$a^3+a^2b+2a^2b+2ab^2+ab^2+b^3$

$= a^3+3a^2b+3ab^2+b^3$

5. $(a+b)^4$

$(a+b)(a^3+3a^2b+3ab^2+b^3)$

$a^4+3a^3b+3a^2b^2+ab^3$
$+a^3b+3a^2b^2+3ab^3+b^4$

$=a^4+4a^3b+6a^2b^2+4ab^3+b^4$

6. What if I wanted to find $(a+b)^9$?

*What patterns do you notice?

- "a" exponent starts high & decreases by 1 each term

- "b" exponent starts at zero & increases by 1 each term

- terms degree is always the overall power

- coefficients follow Pascal's Δ

Pascal's Triangle

power

a^0 1

1 1 1

2 1 2 1

3 1 3 3 1

4 1 4 6 4 1

5 1 5 10 10 5 1

6 1 6 15 20 15 6 1

7 1 7 21 35 35 21 7 1

8 1 8 28 56 70 56 28 8 1

9 1 9 36 84 126 126 84 36 9 1

10 1 10 45 120 210 252 210 120 45 10 1

Use Pascal's Triangle to fully expand the following.

1. $(x+y)^7$

$x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$

2. $(x^2+2y)^6$

$(x^2)^6 + 6(x^2)^5(2y) + 15(x^2)^4(2y)^2 + 20(x^2)^3(2y)^3 + 15(x^2)^2(2y)^4 + 6(x^2)(2y)^5 + (2y)^6$

$= x^{12} + 12x^{10}y + 60x^8y^2 + 160x^6y^3 + 240x^4y^4 + 192x^2y^5 + 64y^6$

3. $(a-5)^3$

$a^3 + 3(a)^2(-5)^1 + 3(a)(-5)^2 + (-5)^3$

$= a^3 - 15a^2 + 75a - 125$

4. $(3x-2y)^5$

$(3x)^5 + 5(3x)^4(-2y) + 10(3x)^3(-2y)^2 + 10(3x)^2(-2y)^3 + 5(3x)^1(-2y)^4 + (-2y)^5$

$= 243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$

5. $(a+3b)^6$

$(a)^6 + 6(a)^5(3b) + 15(a)^4(3b)^2 + 20(a)^3(3b)^3 + 15(a)^2(3b)^4 + 6(a)(3b)^5 + (3b)^6$

$15(9)$ $\quad 20(27)$ $\quad 15(81)$

$= a^6 + 18a^5b + 135a^4b^2 + 540a^3b^3 + 1215a^2b^4 + 1458ab^5 + 729b^6$

26

Practice - Expand using Pascal's Triangle:

1. $(2x + y)^6$

$(2x)^6 + 6(2x)^5(y) + 15(2x)^4(y)^2 + 20(2x)^3(y)^3$
$+ 15(2x)^2(y)^4 + 6(2x)(y)^5 + y^6$

$= 64x^6 + 192x^5y + 240x^4y^2 + 160x^3y^3$
$+ 60x^2y^4 + 12xy^5 + y^6$

2. $(3a - 5b)^4$

3. $(x^2 - 3y)^4$

4. $(a^3 + b^2)^2$

5. $(x + \frac{1}{x})^6$

$x^6 + 6(x)^5(\frac{1}{x}) + 15(x)^4(\frac{1}{x})^2 + 20(x)^3(\frac{1}{x})^3 + 15(x)^2(\frac{1}{x})^4$
$+ 6(x)(\frac{1}{x})^5 + (\frac{1}{x})^6$

$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

6. $(2x - 3)^3$

Find the indicated term of each expression.

7. Fourth term of $(x+2)^7$ 1, 7, 21, 35, 35, 21, 7, 1

$35(x)^4(2)^3 = 280x^4$

8. Sixth term of $(x-y)^9$ 1, 9, 36, 84, 126, 126, 84, 36, 9, 1

$126(x)^4(-y)^5$
$= -126x^4y^5$

9. Fifth term of $(2x+3y)^9$

$126(2x)^5(3y)^4$
$= 326592x^5y^4$

10. Find the term containing y^8 in the expansion of $(2x+3y^2)^9$

$(3y^2)^4$

$126(2x)^5(3y^2)^4$
$= 326592x^5y^8$

11. The middle term in $(3x - 4)^8$

$70(3x)^4(-4)^4$

$= 1451520x^4$

12. Find the 5th term in the expansion of $(3a^3 - 2b^2)^8$.

$70(3a^3)^4(-2b^2)^4$

$= 90720a^{12}b^8$

27