AFM Logs & Exponentials Study Guide

Name key

Simplify:

1. $\left(2^{\sqrt{2}}\right)^{\sqrt{24}}$

$2^{\sqrt{48}}$ $\quad 48 = 2\cdot 24 = 4\cdot 6 = 2\cdot 2\cdot 2\cdot 3$

$\boxed{2^{4\sqrt{3}}}$

2. $13^{\sqrt{6}} \cdot 13^{\sqrt{24}}$

$24 = 4\cdot 6 = 2\cdot 2\cdot 2\cdot 3$ $\quad \sqrt{24} = 2\sqrt{6}$

$\sqrt{6} + 2\sqrt{6} = 3\sqrt{6}$

$\boxed{13^{3\sqrt{6}}}$

3. $\frac{3^{2x-4}}{3^{8-4x}}$

$2x-4-(8-4x)$

$2x-4-8+4x$

$6x-12$

$\boxed{3^{6x-12}}$

4. $\left(n^{\sqrt{3}}m^{-\sqrt{5}}\right)^{\sqrt{5}}$

$n^{\sqrt{15}}m^{-5} = \boxed{\frac{n^{\sqrt{15}}}{m^5}}$

5. $b^{3m-5}\cdot b^{4-m}$

$3m-5+4-m$

$2m-1$

$\boxed{b^{2m-1}}$

6. $\left(\frac{8a^{-2}b^3}{-3b^{-5}}\right)^{-2}\left(\frac{-2b^{-1}}{3a^{14}}\right)^0$

$\frac{8^{-2}a^4b^{-6}}{(-3)^{-2}b^{10}} = \frac{9a^4}{64b^{16}} = \boxed{\frac{9a^4}{64b^{16}}}$

7. $\left(a^{\sqrt{2}}b^{\sqrt{32}}\right)^{\sqrt{2}}$

$a^{\sqrt{4}}b^{\sqrt{64}} = \boxed{a^2b^8}$

8. $\left(m^{\frac{2}{3}}n^{\frac{3}{4}}\right)^2$

$m^{4/3}n^{6/4}$ or $m^{4/3}n^{3/2}$

9. $-2\sqrt[3]{3} + 3\sqrt[3]{81}$

$81 = 9\cdot 9 = 3\cdot 3\cdot 3\cdot 3$

$-2\sqrt[3]{3} + 9\sqrt[3]{3} = \boxed{7\sqrt[3]{3}}$

10. $-2\sqrt{10}\cdot 2\sqrt{6}$

$-4\sqrt{60}$

$60 = 6\cdot 10 = 2\cdot 3\cdot 2\cdot 5$

$= \boxed{-8\sqrt{15}}$

11. $\sqrt[4]{48x^2y^8}$

$48 = 4\cdot 12 = 2\cdot 2\cdot 4\cdot 3 = 2\cdot 2\cdot 2\cdot 2\cdot 3$

XX

YYYY | YYY

$\boxed{2y^2\sqrt[4]{3x^2}}$

12. $\sqrt{192x^3y^4}$

$192 = 4\cdot 48 = 4\cdot 4\cdot 12 = 4\cdot 4\cdot 4\cdot 3 = 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 3$

XXX

YYYY

$\boxed{8xy^2\sqrt{3x}}$

Evaluate without a calculator.

13. $8^{\log_8 3} = 3$

$\log_8 ? = \log_8 3$

14. $\log_a b^4 \cdot \log_b a^5$

not possible

15. $2^{3\log 5 + 3\log 2}$

$2^{\log 1000}$

$2^3 = 8$

$\log 1000 = 3$

16. $\log_3(\log_3(\log_3 27))$ $3^? = 27$

$\log_3(\log_3 3)$ $3^? = 3$

$\log_3(1)$ $3^? = 1$

$= 0$

17. $\frac{4}{5}\log_2 32 - \log_2 8$

$\log_2\left(\frac{16}{8}\right)$

$\log_2 2 = 1$

18. $e^{4\ln 2} = 16$

$\ln ? = \ln 16$

19. $\ln\sqrt[5]{e^2}$

$= 2/5$

20. $\ln e$

$= 1$

True or False:

21. $\ln(x+y) = \ln x + \ln y$ $\ln xy$

False

22. $\log_3\left(\frac{m^2}{3n}\right) = 2\log_3 m - 1 - \log_3 n$

$2\log_3 m - \log_3 3 - \log_3 n$

True

23. $e^{2\ln 3 - 4\ln 2} = \frac{9}{16}$

$e^{\ln \frac{9}{16}}$

True

24. *If* $\log_5(2x-3) = 5$ *then* $x = 4$

$\log_5(2\cdot 4 - 3)$

$= \log_5(5) = 5$

False

$5^5 \neq 5$

25. $\ln 0 = 1$

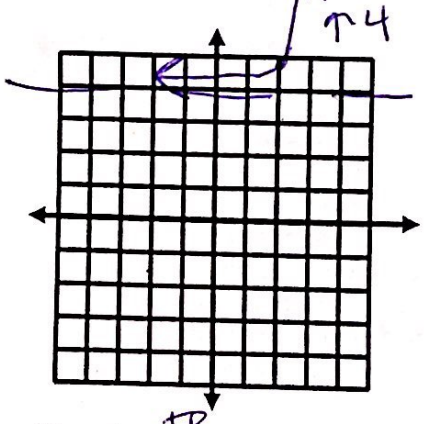
$e^1 \neq 0$ False

26. $\log_3 \frac{1}{243} = -5$

$3^{-5} = \frac{1}{243}$ True

Graph the following and give the domain and range and state asymptotes.

27. $y = 3^{x-2} + 4$ R 2 ↑ 4

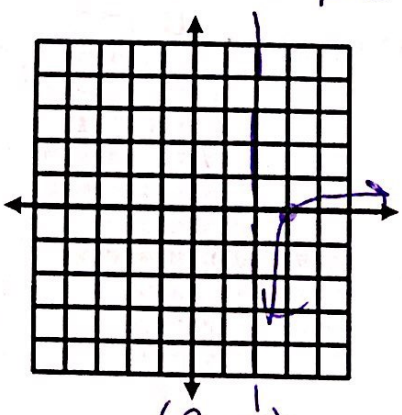


Domain: $\mathbb{R}$

Range: $(4, \infty)$

Asymp: $y = 4$

28. $y = \log(x-2)$ R 2

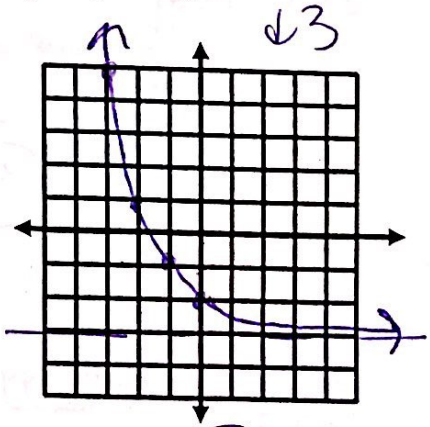


Domain: $(2, \infty)$

Range: $\mathbb{R}$

Asymp: $x = 2$

29. $y = 2^{-x} - 3$ Reflect ↓ 3



Domain: $\mathbb{R}$

Range: $(-3, \infty)$

Asymp: $y = -3$

Solve the following. Exact value or round to 4 places.

30. $\log x + \log(x+21) = 2$

$\log(x^2+21x) = 2$

$10^2 = x^2+21x$

$0 = x^2+21x-100$

$0 = (x+25)(x-4)$

$\boxed{x = -25, 4}$

31. $\log_3(x^2-1) - \log_3(5x+5) = 0$

$\log_3\left(\frac{x^2-1}{5x+5}\right) = 0$

$3^0 = \frac{x^2-1}{5x+5}$

$5x+5 = x^2-1$

$0 = x^2-5x-6$

$(x-6)(x+1) = 0$

$\boxed{x = -1, 6}$

32. $\log_5(3x+7) = 2$

$25 = 3x+7$

$18 = 3x$

$\boxed{6 = x}$

33. $2\log_6 4 - \frac{1}{4}\log_6 16 = \log_6 x$

$\log_6\left(\frac{16}{2}\right) = \log_6 x$

$\boxed{8 = x}$

34. $\log_7(4x+4) = \log_7(3x+10)$

$4x+4 = 3x+10$

$\boxed{x = 6}$

35. $3^{4x} = 42$

$\log_3(42) = 4x$

$4x = 3.4022$

$\frac{4}{4}$

$\boxed{x = .8505}$

36. $4^{x+3} + 2 = 25.8$

$4^{x+3} = 23.8$

$\log_4(23.8) = x+3$

$\boxed{x = -.7136}$

37. $2^{x+1} = 3^{2-2x}$

$(x+1)\log 2 = (2-2x)\log 3$

$x\log 2 + \log 2 = 2\log 3 - 2x\log 3$

$x\log 2 + 2x\log 3 = 2\log 3 - \log 2$

$1.2552x = .6532$

$\boxed{x = .5204}$

38. $250 = 35e^{-.03x}$

$\frac{50}{7} = e^{-.03x}$

$\ln\left(\frac{50}{7}\right) = -.03x$

$\frac{-.03}{-.03}$

$\boxed{x = -65.5371}$

39. $5 - 6e^{4x} = 17$

$-6e^{4x} = 12$

$e^{4x} = -2$

$\ln(-2) = 4x$

$\frac{4}{4}$

$\boxed{\text{No solution}}$

40. $\ln 2x^2 + \ln 8 = 4$

$\ln(16x^2) = 4$

$e^4 = 16x^2$

$\frac{e^4}{16}$

$\boxed{x = \pm 1.8473}$

41. $\log_{15}(x+2) + \log_{15} x = 1$

$\log_{15}(x^2+2x) = 1$

$15^1 = x^2+2x$

$0 = x^2+2x-15$

$0 = (x+5)(x-3)$

$\boxed{x = -5, 3}$

42. Write the equation that transforms $y = \ln x$ with the following transformations: reflect over the y axis, shift left 8 units and shift down 10 units, and stretched vertically by 6.

$y = 6\ln(-(x+8)) - 10$

Set up and solve the following problems involving compound interest. Show all work.

43. How much money will you have in 8 years if you invest $4000 at 3.5% compounded quarterly?

$A = 4000\left(1 + \frac{.035}{4}\right)^{4(8)}$

$A = \$5286.08$

44. How much money do you need to invest at 2.8% compounded continuously in order to have $25,500 at the end of 8 years?

$25500 = Pe^{.028(8)}$

$\frac{25500}{1.25107} = P(1.25107)$

$P = \$20,382.54$

45. How much money will you have in 6 months if you invest $1000 at 3% compounded monthly?

$A = 1000\left(1 + \frac{.03}{12}\right)^{12(.5)}$

$A = \$1015.09$

46. How much interest will you earn in 8 years if you invest $7500 at 4 ¼ % compounded semi-annually?

$A = 7500\left(1 + \frac{.0425}{2}\right)^{2(8)}$

$A = 10,499.64$

Interest earned $= \$2999.64$

47. If you deposit $4500 at 5% annual interest compounded continuously, how much money will be in the account after 10 years?

$A = 4500e^{.05(10)}$

$A = \$7419.25$

48. How long will it take you to double your money in an account that earns 4.6% compounded monthly?

$200 = 100\left(1 + \frac{.046}{12}\right)^{12t}$

$2 = 1.00383^{12t}$

$\frac{\log_{1.00383}(2)}{12} = 12t$

$t = 15.097$ years