Operations with Radicals

$\sqrt[b]{a}$

Vocabulary:

*Index: b → # outside radical

*Radical: symbol

*Radicand: a → term(s) under the radical

To simplify radicals....

Make a factor tree! Whatever the index is, that is how many you need in order to take that number/variable out of the radical.

wwwwwwww

Examples:

a. $\sqrt{150x^7}$

15 · 10

3 · 5 · 5 · 2

x x x x x x x

$\boxed{5x^3\sqrt{6x}}$

b. $\sqrt[3]{-64w^8}$

8 · 8

4 · 2 4 · 2

2 · 2 2 · 2

$\boxed{-4w^2\sqrt[3]{w^2}}$

c. $-2\sqrt{112xy^3}$

2 · 56

8 · 7

4 · 2

2 · 2

x y y y

$\boxed{-8y\sqrt{7xy}}$

d. $5\sqrt{32x^4y^3}$

8 · 4

4 · 2 2 · 2

2 · 2

x x x x

y y y

$\boxed{20x^2y\sqrt{2y}}$

Practice:

a. $\sqrt{45x^6}$

9 · 5

3 · 3

x x x x x x

$\boxed{3x^3\sqrt{5}}$

b. $\sqrt[3]{-108a^4b^2}$

2 · 54

9 · 6

3 · 3 3 · 2

a a a a

b b

$\boxed{-3a\sqrt[3]{4ab^2}}$

c. $-2\sqrt[4]{486a^6b^7}$

2 · 243

3 · 81

9 · 9

3 · 3 3 · 3

a a a a a a

b b b b b b b

$\boxed{-6ab\sqrt[4]{6a^2b^3}}$

d. $2\sqrt[3]{250m^7n^5}$

25 · 10

5 · 5 2 · 5

m m m m m m m

n n n n n

$\boxed{10m^2n\sqrt[3]{2mn^2}}$

1

…iplying Radicals

…hen written in radical form, it's only possible to write two multiplied radicals as one if the index is the same.
As long as this requirement is met,

1) multiply the Outsides
2) multiply the Insides
3) Simplify!

Let's try some examples…

$2\sqrt{3}\cdot 5\sqrt{2}$ $10\sqrt{6}$	$-3\sqrt{8}\cdot\sqrt{2}$ $-3\sqrt{16}$ $=-3(4)=\boxed{-12}$	$4\sqrt{5}\cdot 3\sqrt{10}$ $12\sqrt{50}$ 50 10 · 5 2 5 $=60\sqrt{2}$
$\sqrt{3x^2y}\cdot\sqrt{5xy}$ xxx yy $\sqrt{15x^3y^2}$ $xy\sqrt{15x}$	$6\sqrt{8x^3y^2}\cdot\sqrt{10xy^3}$ 80 8 · 10 4 2 2 5 22 $6\sqrt{80x^4y^5}$ xxxx yyyyy $\boxed{24x^2y^2\sqrt{5y}}$	$-\sqrt{5x^4y^3}\cdot\sqrt{15x^2y^5}$ 75 3 · 25 5 · 5 $-1\sqrt{75x^6y^8}$ $\boxed{-5x^3y^4\sqrt{3}}$

NOTES: ADDING AND SUBTRACTING RADICALS

You've been combining like terms in algebraic expressions for a long time! Show your skills by simplifying the following expressions.

$2x - x + 4x =$ 5x

$3y - 2x + y - 6y =$ $-2x-2y$

Usually we say that like terms are those that contain the same variable expression, but they can also contain the same radical expression. *When you add or subtract radicals, you can only do so if they contain the same index and radicand.* Just like we don't change the variable expression when we add or subtract, we're not going to change the radical expression either. All we are going to do is add or subtract the coefficients.

★Always simplify the radical before you decide that you can't add or subtract.

Let's try some examples....

$3\sqrt{3} + 4\sqrt{3}$ $7\sqrt{3}$	$\sqrt{5} + 2\sqrt{5} + 3\sqrt{5}$ $6\sqrt{5}$	$4\sqrt{12} - \sqrt{75}$ 3·4 3·25 2·2 5·5 $8\sqrt{3} - 5\sqrt{3} = \boxed{3\sqrt{3}}$
$\sqrt{45x^3} - \sqrt{20x^3}$ x x x 9·5 4·5 3·3 2·2 $3x\sqrt{5x}\ 2x\sqrt{5x} = x\sqrt{5x}$	$5\sqrt[3]{32} - 2\sqrt[3]{108}$ 4·8 108 2·2 4·2 2·54 2 2 9·6 3·3 3 2 $10\sqrt[3]{4} - 6\sqrt[3]{4}$ $= 4\sqrt[3]{4}$	$3\sqrt[3]{16} + \sqrt[3]{54}$ 4·4 9·6 2 2 2 2 3 3 3 2 $6\sqrt[3]{2} + 3\sqrt[3]{2}$ $= 9\sqrt[3]{2}$

GROUP WORK

1. a. $(-2\sqrt{7})(3\sqrt{35})$

b. $3\sqrt{15} \cdot 2\sqrt{75}$

c. $9\sqrt{3} + 2\sqrt{3}$

d. $3\sqrt{7} - 7\sqrt[3]{7}$

e. $\sqrt{p^3} \cdot \sqrt{p^5}$

f. $\sqrt{2ab^2} \cdot \sqrt{8a^3b} \cdot \sqrt{3b^3c^6}$

g. $3\sqrt{32} + 2\sqrt{50}$

h. $\sqrt{200} - \sqrt{72}$

i. $14\sqrt[3]{xy} - 3\sqrt[3]{xy}$

_1 – Exponent Rules Review

Name ______________________

_es of Exponents:

- When multiplying like bases, you Add exponents
- When dividing like bases, you subtract exponents
- When you raise a power to a power, you multiply exponents
- To fix negative exponents, you flip it + make it positive
- Anything raised to the zero power is ~~zero~~ one.

1. $2(-3.5)^0$

$2(1) = \boxed{2}$

2. $7^{-1} \cdot 10^0$

$\frac{1}{7} \cdot 1 = \boxed{\frac{1}{7}}$

3. $3^5 \cdot 3^{-8} \cdot 3^6$

$3^5 \cdot \frac{1}{3^8} \cdot 3^6$

$= \frac{3^{11}}{3^8} = \boxed{3^3 = 27}$

4. $8m^{-4}n^{-4}$

$\frac{8}{m^4n^4}$

5. $7x^{-3} \cdot 4x^5$

$\frac{7}{x^3} \cdot \frac{4x^5}{1}$

$= \boxed{28x^2}$

6. $(x^8)^3(x^{-4})^0$

x^{24}

7. $(d^3)^{-2}$

d^{-6}

$= \boxed{\frac{1}{d^6}}$

8. $\frac{9}{a^5b^{-3}}$

$\frac{9b^3}{a^5}$

9. $18 \cdot 2^{-2}$

$18 \cdot \frac{1}{2^2}$

$= \frac{18}{4} = \boxed{\frac{9}{2}}$

10. $(2k^4)^3$

$\boxed{8k^{12}}$

11. $(-5x^5) \cdot 3y^5 \cdot 7x^5$

$\boxed{-105x^{10}y^5}$

12. $4^1 \cdot 4^{3x} \cdot 4^{x+3}$

4^{4x+4}

13. $(4g^8)^3$

$64g^{24}$

14. $(-3a^3b^4)^3(a^5b^5)^5$

$-27a^9b^{12} \cdot a^{25}b^{25}$

$= \boxed{-27a^{34}b^{37}}$

15. $\frac{c^{12}}{c^4}$

c^8

16. $\frac{8^7}{8^9}$

$\frac{1}{8^2} = \frac{1}{64}$

17. $\frac{t^5}{t^9}$

$\frac{1}{t^4}$

18. $\frac{3^6}{3^9}$

$\frac{1}{3^3} = \frac{1}{27}$

19. $\frac{m^7n^9}{m^{20}n^{-2}}$

$\frac{n^{11}}{m^{13}}$

20. $\left(\frac{2c}{4a}\right)^3$

$\frac{8c^3}{64a^3} = \frac{c^3}{8a^3}$

Rational Exponents

Definition of Rational Exponents For any nonzero number b, and any integers m and n with n>1,

$$b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$$

Power
Root

1. Evaluate.

a. $625^{\frac{1}{4}}$

$\sqrt[4]{625} = 5$

$\sqrt[4]{5^4} = 5$

b. $125^{\frac{1}{3}}$

$\sqrt[3]{125} = 5$

c. $3^{\frac{1}{2}} \cdot 21^{\frac{1}{2}}$

$\sqrt{3} \cdot \sqrt{21}$

$= \sqrt{63}$

$9 \cdot 7 = 3\sqrt{7}$

$3\ 3$

d. $4^{\frac{1}{3}} \cdot 16^{\frac{1}{3}}$

$\sqrt[3]{4} \cdot \sqrt[3]{16} = \sqrt[3]{64} = 4$

e. $81^{\frac{5}{4}}$

$\sqrt[4]{81^5} = 3^5 = 243$

$9 \cdot 9$

$3\ 3\ 3\ 3$

f. $8^{\frac{2}{3}}$

$\sqrt[3]{8^2}$

$= 2^2 = 4$

g. $27^{\frac{2}{3}}$

$\sqrt[3]{27^2} = 3^2 = 9$

$9 \cdot 3$

$3 \cdot 3$

2. Express using rational exponents.

a. $\sqrt{23}$

$23^{1/2}$

b. $\sqrt[3]{63}$

$63^{1/3}$

c. $\sqrt[4]{16z^2}$

$16^{1/4} z^{2}$

d. $\sqrt[3]{5x^2y}$

$5^{1/3} x^{2/3} y^{1/3}$

e. $\sqrt[4]{27x^4y^3}$

$27^{1/4} x^{1} y^{3/4}$

3. Express using radicals.

a. $6^{\frac{1}{5}}$

$\sqrt[5]{6}$

b. $4^{\frac{1}{3}}$

$\sqrt[3]{4}$

c. $c^{\frac{2}{5}}$

$\sqrt[5]{c^2}$

d. $(x^2)^{\frac{4}{3}} = x^{8/3}$

$\sqrt[3]{x^8}$

e. $(5a)^{\frac{2}{3}} b^{\frac{5}{3}}$

$\sqrt[3]{25a^2b^5}$

4. Simplify.

a. $y^{\frac{5}{3}} y^{\frac{7}{3}}$

$y^{12/3} = y^4$

b. $(b^{\frac{1}{3}})^{\frac{3}{5}}$

$b^{\frac{3}{15}} = b^{\frac{1}{5}}$

c. $\sqrt[3]{a^4b^8}$

$ab^2\sqrt[3]{ab^2}$

d. $\sqrt{8m^5n^4}$

$4 \cdot 2$

$2 \cdot 2$

$2m^2n^2\sqrt{2m}$

e. $\sqrt[4]{32a^9b^{11}}$

$4 \cdot 8$

$2\ 2\ 4\ 2$

$2\ 2$

$2a^2b^2\sqrt[4]{2ab^3}$

f. $y^{\sqrt{5}} y^{\sqrt{5}}$

$y^{2\sqrt{5}}$

g. $\dfrac{(2-\sqrt{5})^{x+1}}{(2-\sqrt{5})^{x-1}}$

$(x+1)-(x-1)$

$= x+1-x+1$

$= 2$

$= (2-\sqrt{5})^2$

h. $\dfrac{4^{\sqrt{3}} \bullet 4^{\sqrt{18}}}{4^{3\sqrt{3}}}$

$\dfrac{4^{\sqrt{3}} \cdot 4^{3\sqrt{2}}}{4^{3\sqrt{3}}}$

18

$9 \cdot 2$

$3\ 3$

$\sqrt{3} - 3\sqrt{3} = -2\sqrt{3}$

$\dfrac{4^{3\sqrt{2}}}{4^{2\sqrt{3}}} = \boxed{4^{3\sqrt{2}-2\sqrt{3}}}$ 7

...ponential Functions

...xponential Functions have the form $y = a \cdot b^x$ where b is a positive real number.

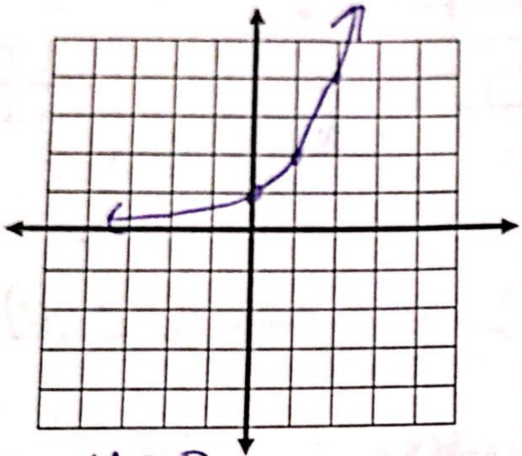
a = Initial value b = base

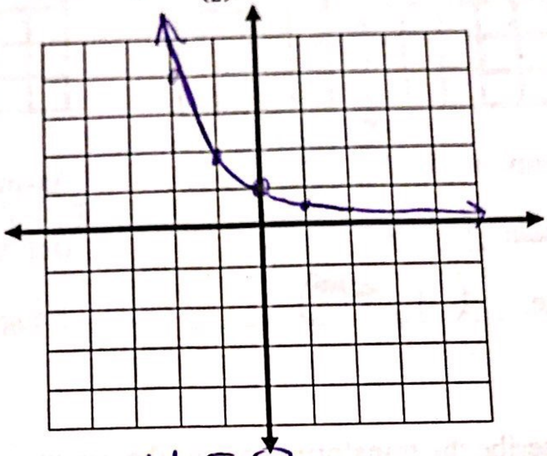
Transformation form: $y = a(2)^{b(x-c)} + d$

a: V. stretch/comp b: H. stretch/comp

c: R/L d: up/↓

1. Graph and compare the graphs of $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$





Asymptote: y=0 Asymptote: y=0

Domain: $(-\infty,\infty)$ Domain: $(-\infty,\infty)$

Range: $(0,\infty)$ Range: $(0,\infty)$

Describe the transformations for the following:

2. Parent function: $y=2^x$

a) $y = 3(2)^x$ V. stretch by 3

b) $y = 2^{\frac{x}{4}}$ H stretch by 4

c) $y = 2^x + 3$ up 3

d) $y = 2^{x-1}$ R1

3. Parent function: $y=\frac{1}{2}^x$

a) $y = 2\left(\frac{1}{2}\right)^x - 4$ V. stretch by 2 ↓4

b) $y = \left(\frac{1}{2}\right)^{x+3} - 2$ L3, ↓2

c) $y = \left(\frac{1}{2}\right)^{\frac{1}{2}(x-1)}$ H stretch by 2, R1

d) $y = \left(\frac{1}{2}\right)^{2x+6}$

$\frac{1}{2}^{2(x+3)}$

L3

H comp $\frac{1}{2}$

Scanned with CamScanner

4. Describe the transformations of the parent graph $y = (\frac{1}{2})^x$ then graph below.

a. $y = \left(\frac{1}{2}\right)^x + 1$

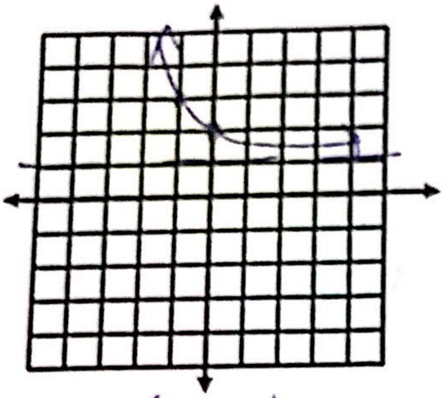
b. $y = -\left(\frac{1}{2}\right)^x$

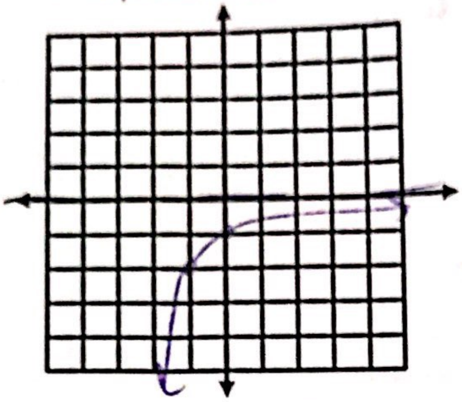
c. $y = -\left(\frac{1}{2}\right)^{x+4}$

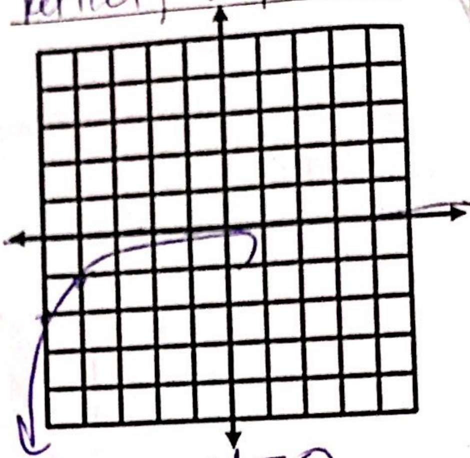
a. up 1

b. Reflect x axis

c. Reflect, L4







Asymptote: y=1

Domain: $(-\infty,\infty)$

Range: $(1,\infty)$

Asymptote: y=0

Domain: $(-\infty,\infty)$

Range: $(-\infty,0)$

Asymptote: y=0

Domain: $(-\infty,\infty)$

Range: $(-\infty,0)$

5. Describe the transformations of the parent graph $y = 3^x$ then graph below.

a. $y = (3)^x + 2$

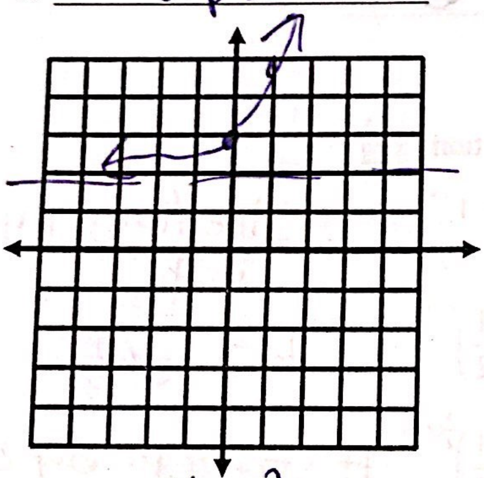
b. $y = 2(3)^x - 3$

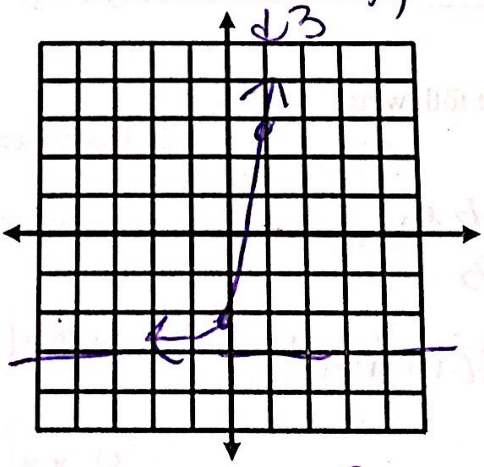
c. $y = -(3)^{x-1}$

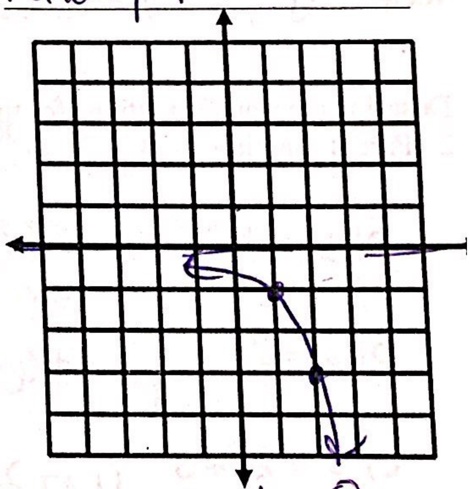
a. up 2

b. V. Stretch by 2, d3

c. Reflect, R1







Asymptote: y=2

Domain: $(-\infty,\infty)$

Range: $(2,\infty)$

Asymptote: y=-3

Domain: $(-\infty,\infty)$

Range: $(-3,\infty)$

Asymptote: y=0

Domain: $(-\infty,\infty)$

Range: $(-\infty,0)$

Evaluate the following without a calculator for $f(x) = 3^x$

a. $f(2)$ b. $f(-3)$ c. $f(\frac{4}{3})$ d. $f(0)$ $3^0=1$

$3^2=9$

$3^{-3}=\frac{1}{3^3}=\frac{1}{27}$

$3^{4/3}$

$=\sqrt[3]{3^4}=\sqrt[3]{81}=3\sqrt[3]{3}$

The Number e

An irrational number, symbolized by the letter e, appears as the base in many applied exponential functions. This irrational number is approximately equal to 2.72.

Evaluate $y = (1+\frac{1}{n})^n$ when n = 10, n=100, n = 1000,.... Euler's #

As n approaches infinite, y converges to a number called e

Round e to 4 places: 2.7183

6. Simplify without a calculator.

a. $e^4 \cdot e^{-5}$

$e^{-1}=\frac{1}{e}$

b. $\sqrt[3]{64e^3}$ eee

$8\cdot 8$

$4\ 2\ 4\ 2$

$2\ 2\ 2\ 2$

$\boxed{4e}$

c. $\frac{12e^{-2}}{18e^{-7}}$

$\frac{2e^7}{3e^2}$

$=\frac{2e^5}{3}$

d. e^0

1

Solving exponential equations with a change of base:

*Get bases of exponents to be the same

*Once the same, bases can be dropped

*Solve the exponents

7. a. $3^x = 27$

$\cancel{3}^x=\cancel{3}^3$

$\boxed{x=3}$

b. $\cancel{5}^{2x+3} = \cancel{5}^{x+4}$

$2x+3=x+4$

$\boxed{x=1}$

c. $4^x = \frac{1}{16}$

$\cancel{4}^x=\cancel{4}^{-2}$

$\boxed{x=-2}$

d. $3^{n-2} = 27$

$\cancel{3}^{n-2}=\cancel{3}^3$

$n-2=3$

$\boxed{n=5}$

e. $(\frac{1}{9})^m = 81^{m+4}$

$(9^{-1})^m=(9^2)^{m+4}$

$\cancel{9}^{-m}=\cancel{9}^{2m+8}$

$-m=2m+8$

$-3m=8$

$\boxed{m=-8/3}$

f. $\left(\frac{1}{16}\right)^{x+1} = \left(\frac{1}{8}\right)^{2x-1}$

$(2^{-4})^{x+1}=(2^{-3})^{2x-1}$

$\cancel{2}^{-4x-4}=\cancel{2}^{-6x+3}$

$-4x-4=-6x+3$

$2x=7$

$\boxed{x=7/2}$

Logarithm Functions

Definition of a Logarithmic Function:
The logarithmic function $y = \log_b x$ where $b > 0$ and $b \neq 1$ is equivalent to $b^y = x$.

b = base **x =** argument

y = exponent

What are Logarithms??? Exponents!

1. Write in exponential form.

a. $\log_5 125 = 3$

$5^3 = 125$

b. $\log_3 169 = 2$

$13^2 = 169$

c. $\log_4 \frac{1}{4} = -1$

$4^{-1} = \frac{1}{4}$

d. $\log_{\frac{1}{5}} 25 = -2$

$\frac{1}{5}^{-2} = 25$

e. $\log 100 = 2$

$10^2 = 100$

f. $\log \frac{1}{1000} = -3$

$10^{-3} = \frac{1}{1000}$

2. Write in logarithmic form.

a. $8^3 = 512$

$\log_8 512 = 3$

b. $3^3 = 27$

$\log_3 27 = 3$

c. $5^{-3} = \frac{1}{125}$

$\log_5 \frac{1}{125} = -3$

3. Evaluate each expression **without** a calculator.

a. $\log_2 16 = 4$

$2^? = 16$

$2^4 = 16$

b. $\log_{12} 144 = 2$

$12^? = 144$

c. $\log_{16} 4 = \frac{1}{2}$

$16^? = 4$

d. $\log_9 1 = 0$

$9^? = 1$

e. $\log_2 \frac{1}{32} = -5$

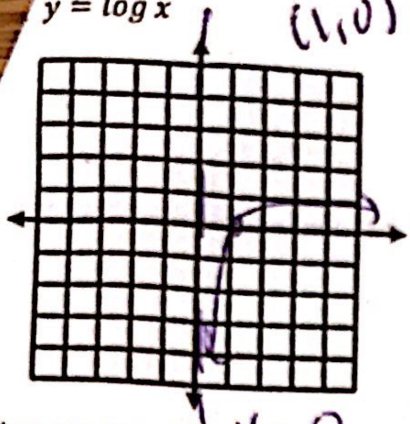
$2^? = \frac{1}{32}$

f. $\log_3 \frac{1}{81} = -4$

$3^? = \frac{1}{81}$

Graph each logarithm.

$y = \log x$ (1,0)



Asymptote: $x=0$

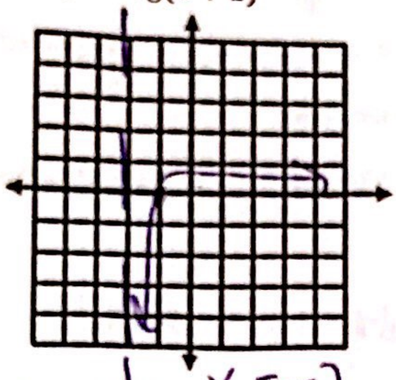
Domain: $(0,\infty)$

Range: $(-\infty,\infty)$

Transformations:

none

b. $y = \log(x+2)$



Asymptote: $x=-2$

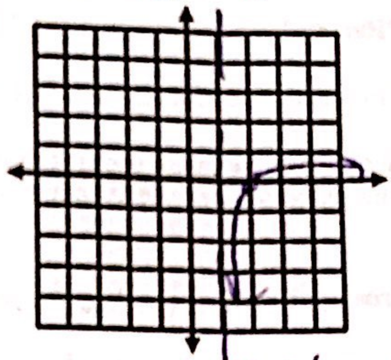
Domain: $(-2,\infty)$

Range: $(-\infty,\infty)$

Transformations:

L 2

c. $y = \log(x-1)$



Asymptote: $x=1$

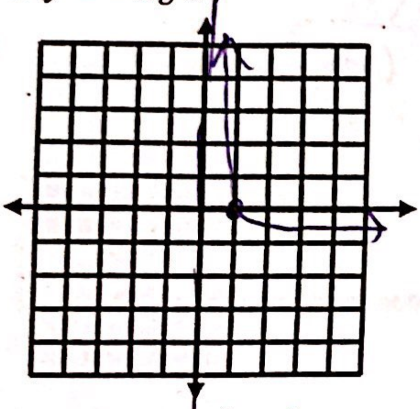
Domain: $(1,\infty)$

Range: $\mathbb{R}$

Transformations:

R1

d. $y = -\log x$



Asymptote: $x=0$

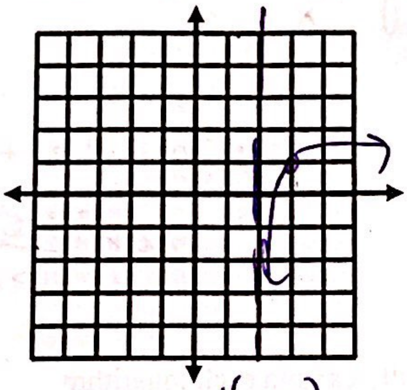
Domain: $(0,\infty)$

Range: $\mathbb{R}$

Transformations:

Reflect x-axis

e. $y = \log(x-2)+1$



Asymptote: $x=2$

Domain: $(2,\infty)$

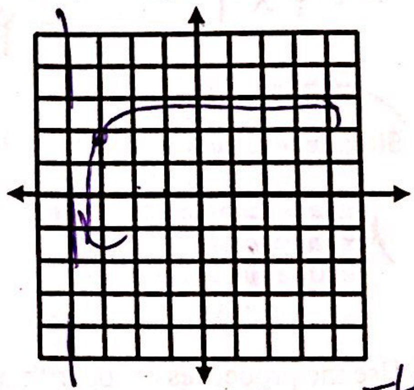
Range: $\mathbb{R}$

Transformations:

R2

↑1

f. $y = \log(x+4)+2$



Asymptote: $x=-4$

Domain: $(-4,\infty)$

Range: $\mathbb{R}$

Transformations:

L4

↑2

Properties of Logs

***Remember rules of exponents? They're the SAME as the rules of logs!**

***In order to condense, bases MUST be the same.**

Suppose m and n are positive numbers, b is a positive number other than 1, and p is any real number. The following properties will be true:

Product Property: $\log mn =$ $\log m + \log n$

Quotient Property: $\log \frac{m}{n} =$ $\log m - \log n$

Power Property: $\log m^n =$ $n \log m$

Property of Equality: If $\log m = \log n$ then

5. Write the expression as a single logarithm.

a. $\log x + 3\log y$

$\log xy^3$

b. $\log(2x+5) - \log x$

$\log \frac{2x+5}{x}$

c. $\frac{1}{2}(\log_5 x + \log_5 y) - 2\log_5(x+1)$

$\log \frac{\sqrt{xy}}{(x+1)^2}$

d. $5\log_2 x - 4\log_2 y$

$\log \frac{x^5}{y^4}$

e. $\frac{1}{3}\log a - \log b + 4\log c$

$\log \frac{\sqrt[3]{a}\,c^4}{b}$

6. Use the properties of logarithms to fully expand each logarithm.

a. $\log_9 9x$

$\log_9 9 + \log_9 x$

b. $\log_4\left(\frac{64}{y}\right)$

$\log_4 64 - \log_4 y$

c. $\log_b\left(\frac{x^2y}{z^2}\right)$

$2\log_b x + \log_b y$
$- 2\log_b z$

d. $\log\frac{x+2}{y}$

$\log(x+2) - \log y$

e. $\log_2 a^3\sqrt{bc}$

$3\log_2 a + \frac{1}{2}\log_2 b$
$+ \frac{1}{2}\log_2 c$

Evaluate each **without** a calculator.

a. $\log_2 8$

$2^? = 8 = 3$

b. $\log \frac{1}{1000}$ $= -3$

c. $log_4 4^x$

x

d. $log_7 7^8$

8

e. $\log_3 (\log_4 64)$ $4^? = 64$

$\log_3(3) = 1$

$3^? = 3$

f. $3^{log_3 17}$ $= 17$

$\log_3 ? = \log_3 17$

g. $\frac{1}{2}\log_5 16 - 2\log_5 10$

$\log_5 \sqrt{16} - \log_5 10^2$

$= \log_5 \left(\frac{4}{100}\right)$

$\log_5 \left(\frac{1}{25}\right) = -2$

h. $\frac{1}{3}(\log_2 12 + \log_2 16 - \log_2 3)$

$\log_2 \left(\frac{12(16)}{3}\right)^{1/3}$

$\log_2 (64)^{1/3}$

$\log_2 4 = 2$

Logarithms Homework

Write each equation in logarithmic form.

1. $2^5 = 32$ 2. $5^{-3} = \frac{1}{125}$ 3. $6^{-3} = \frac{1}{216}$

Solving Logs

Steps:

1. Condense both sides to a single log if possible.
2. a) If 1 log in total, then change to exponential form

   b) If a log = log with the same base, then drop the bases and solve
3. Plug solutions back in to make sure nothing becomes negative or zero (extraneous solutions)

Solve the following. Show all work.

a. $\log_b 5 = -\frac{1}{3}$

$(b^{-1/3})^{-3} = (5)^{-3}$

$b = \frac{1}{125}$

b. $\log_3 5 + \log_3 x = \log_3 10$

$\log_3 5x = \log_3 10$

$5x = 10$

$\boxed{x = 2}$

c. $\log 16 - \log 2t = \log 2$

$\log \frac{16}{2t} = \log 2$

$\frac{16}{2t} = 2(2t)$

$16 = 4t$ $\boxed{t = 4}$

d. $\log(2x+5) = \log(5x-4)$

$2x+5 = 5x-4$

$9 = 3x$

$\boxed{3 = x}$

e. $\log_3(4x+5) - \log_3(3-2x) = 2$

$\log_3\left(\frac{4x+5}{3-2x}\right) = 2$

$(3-2x)\,9\ 3^2 = \frac{4x+5}{3-2x}(3-2x)$

$27 - 18x = 4x+5$

$22 = 22x$

$\boxed{x = 1}$

f. $\log_2 n = \frac{1}{4}\log_2 16 + \frac{1}{2}\log_2 49$

$\sqrt[4]{16} = 2$

$\sqrt{49} = 7$

$\log_2 n = \log_2(2 \cdot 7)$

$\boxed{n = 14}$

g. $2\log 6 - \frac{1}{3}\log 27 = \log x$

$\log \frac{36}{3} = \log x$

$\boxed{12 = x}$

h. $\log_6(a^2+2) + \log_6 2 = 2$

$\log_6(2a^2+4) = 2$

$36\ 6^2 = 2a^2+4$

$32 = 2a^2$

$16 = a^2$

$\boxed{\pm 4 = a}$

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$log_3(x-5)+log_3(x+3)=2$

$\log_3(x^2-2x-15)=2$

$9\ \cancel{3^2}=x^2-2x-15$

$0=x^2-2x-24$

$0=(x-6)(x+4)$

$x=6,\cancel{-4}$

extraneous!

j. $\log x+\log(x-3)=1$

$\log(x^2-3x)=1$

$10^1=x^2-3x$

$0=x^2-3x-10$

$0=(x-5)(x+2)$

$x=5,\cancel{-2}\rightarrow$ extraneous

k. $\log_9(2x+16)=\log_9(x^2-4x)$

$2x+16=x^2-4x$

$0=x^2-6x-16$

$0=(x-8)(x+2)$

$\boxed{x=8,-2}$

l. $\log_2 x+\log_2(x-2)=3$

$\log_2(x^2-2x)=3$

$2^3=x^2-2x$

$0=x^2-2x-8$

$0=(x-4)(x+2)$

$x=4,\cancel{-2}\rightarrow$ extraneous

Solving Logs Homework

Solve each equation. Show all work.

1. $\log_2 24-\log_2 2=\log_2 x$

$\log_2\frac{24}{2}=\log_2 x$

$\boxed{x=12}$

2. $3\log_5 2=\log_5 x$

$\boxed{x=8}$

3. $\log_6 x-\log_6 5=\log_6 4$

$\log_6\frac{x}{5}=\log_6 4$

$\frac{x}{5}=4$

$\boxed{x=20}$

4. $\log x=\frac{1}{3}\log 8+\frac{1}{2}\log 81$

$\sqrt[3]{8}=2$

$\sqrt{81}=9$

$\log x=\log(2\cdot 9)$

$\boxed{x=18}$

Solving Logarithm & Exponential Functions

Exponentials:

1. If you have **ONE** exponent, change it to log form. Then solve.
2. If you have **TWO** exponents:
   a. Make the same base if possible
   b. If they can't be the same base, take the log of both sides

1. $9^x = 45$

$x \log 9 = \log 45$

$\log_9 45 = x$

$\boxed{x = 1.732}$

2. $5^x = 52$

$\log_5 52 = x$ or $\frac{\log 52}{\log 5}$

$\boxed{x = 2.455}$

3. $4^{3p} = 10$

$3p \log 4 = \frac{\log 10}{3 \log 4}$

$\frac{\log_4 10 = 3p}{3}$

$\boxed{p = .554}$

4. $5^{-7x-10} + 4 = 37$

$5^{-7x-10} = 33$

$\log_5 33 = -7x - 10$

$\frac{\log_5 33 + 10}{-7} = x$

$\boxed{x = -1.739}$

5. $8 \cdot 10^x + 4 = 4068$

$8 \cdot 10^x = 4064$

$10^x = \frac{4064}{8}$

$\log\left(\frac{4064}{8}\right) = x$

$\boxed{x = 2.706}$

6. $15^{x-6} - 5 = 71$

$15^{x-6} = 76$

$\log_{15} 76 = x - 6$

$x = 7.599$

$\log_{15} 76 + 6 = x$

$\boxed{x = 7.\cancel{627}}$

7. $5^{x-1} = 3^x$

$(x-1)\log 5 = x \log 3$

$x \log 5 - \log 5 = x \log 3$

$x \log 5 - x \log 3 = \log 5$

$x(\log 5 - \log 3) = \underline{\log 5}$

$\boxed{x = 3.151}$

8. $3^{x+1} = 7^{x-2}$

$(x+1)\log 3 = (x-2)\log 7$

$x \log 3 + \log 3 = x \log 7 - 2 \log 7$

$x \log 3 - x \log 7 = -2 \log 7 - \log 3$

$x(\log 3 - \log 7) = \underline{-2 \log 7 - \log 3}$

$\boxed{x = 5.5898}$

2

9. $2^{x-1} = 6^{2x+3}$

$(x-1)\log 2 = (2x+3)\log 6$

$x\log 2 - \log 2 = 2x\log 6 + 3\log 6$

$x\log 2 - 2x\log 6 = 3\log 6 + \log 2$

$x(\log 2 - 2\log 6) = \underline{\ \ d\ \ }$

$\boxed{\cancel{x(\ldots)}}$ $\boxed{x = .117}$

OMIT

10. $\log_7(x + 10) + \log_7 x = 75$

$\log_7(x^2 + 10x) = \cancel{75}$

$7^{75} = x^2 + 10x$

$0 = x^2 + 10x -$

11. $\log_2 x + \log_2(x + 2) = 3$

$\log_2(x^2 + 2x) = 3$

$2^3 = x^2 + 2x$

$0 = x^2 + 2x - 8$

$0 = (x + 4)(x - 2)$

$x = \cancel{-4}, 2$

12. $5a^{\frac{2}{5}} = 15.35$

$(a^{2/5})^{5/2} = (3.07)^{5/2}$

$\boxed{a = \pm 16.514}$

Power / Root → Even ±

13. $3x^{\frac{4}{3}} = 21.3$

$(x^{4/3})^{3/4} = (7.1)^{3/4}$

$\boxed{x = \pm 4.3495}$

14. $4^{2x-1} = 9^{x+2}$

$(2x-1)\log 4 = (x+2)\log 9$

$2x\log 4 - \log 4 = x\log 9 + 2\log 9$

$2x\log 4 - x\log 9 = 2\log 9 + \log 4$

$x(2\log 4 - \log 9) = \underline{\ \ d\ \ }$

$\boxed{x = 10.047}$

The Natural Log

Base of a natural log = e ≈ 2.7183 The natural log of x is written: $\ln x$

*The same properties and rules of logs apply to the natural log!

Express each in log form.

1. $e^{-2} \approx 0.1353$

$\ln .1353 = -2$

2. $e^{m} = n$

$\ln(n) = m$

3. $e^{0} = 1$

$\ln 1 = 0$

Express each in exponential form.

4. $\ln 4.2 = x$

$e^{x} = 4.2$

5. $\ln \frac{1}{4} \approx -1.3863$

$e^{-1.3863} = \frac{1}{4}$

6. $\ln e = 1$

$e^{1} = e$

Evaluate without a calculator.

7. $\ln\sqrt{e} = \frac{1}{2}$

$e^{?} = e^{\frac{1}{2}}$

8. $\ln e^{5} = 5$

$e^{?} = e^{5}$

9. $\ln(-5)$ DNE

$e^{?} = -5$

10. $e^{\ln 4} = 4$

$\ln ? = \ln 4$

e

Use the properties of logarithms to express as a single log.

11. $\ln 48 - 4\ln 2$

$\ln \frac{48}{2^4} = \frac{48}{16}$

$= \ln 3$

12. $\frac{1}{2}\ln 9 + \ln 12 - 2\ln 3$

$\ln 3 + \ln 12 - \ln 9$

$= \ln\left(\frac{3 \cdot 12}{9}\right) = \ln 4$

13. $\frac{1}{3}\ln x + 2\ln y$

$\ln \sqrt[3]{x}\, y^{2}$

Expand using the properties of logarithms.

14. $\ln\left(\frac{2a^7}{b^3}\right)$

$\ln 2 + 7\ln a - 3\ln b$

15. $\ln\left(\frac{1}{ab}\right)$

$\ln 1 - \ln a - \ln b$

16. $\ln \frac{x+y}{x-y}$

$\ln(x+y) - \ln(x-y)$

17. Write the equation of a natural logarithm that has been vertically compressed by 4, reflected across the x-axis, shifted right 6 units, horizontally stretched by 2, and shifted down 3 units.

$y = -\frac{1}{4}\ln\left(\frac{1}{2}(x-6)\right) - 3$

2

Evaluate without a calculator. Leave answers in exact value form.

18. lne^5 $=5$

19. $ln\frac{1}{e}$ $=-1$

$e^{?}=\frac{1}{e}$

20. $ln\sqrt[7]{e^4}$ $=4/7$

$e^{?}=e^{4/7}$

21. $e^{ln6+ln5}$

$e^{ln30}=30$

$e^{ln?}=ln30$

22. $e^{lnx}=12$

$ln12=lnx$

$x=12$

23. $\frac{1}{2}ln4+ln8-(5ln2+ln3)$

$ln2+ln8-(ln32\cdot 3)$

$ln\frac{16}{96}=\boxed{ln\frac{1}{6}}$

Solve with a calculator. Round to 3 decimal places if necessary.

24. $220=100e^{0.06x}$

$2.2=e^{.06x}$

$\frac{ln2.2=.06x}{.06}$

$\boxed{x=13.141}$

25. $\ln(x-1)-ln8=2$

$ln\frac{x-1}{8}=2$

$e^2=\frac{x-1}{8}$

$8e^2+1=x$

$\boxed{x\approx 60.112}$

26. $\ln(x-4)+ln10=5$

$ln(10x-40)=5$

$e^5=10x-40$

$\frac{e^5+40}{10}=x$

$\boxed{x=18.841}$

27. $1.2e^{-5x}+2.6=3$

$1.2e^{-5x}=.4$

$e^{-5x}=\frac{1}{3}$

$\frac{ln\frac{1}{3}=-5x}{-5}$

$\boxed{x=.2197}$

28. $\frac{10}{1+2e^{-4x}}=9$

$\frac{10}{9}=1+2e^{-4x}$

$\frac{1}{9}=2e^{-4x}$

$\frac{1}{18}=e^{-4x}$

$\frac{ln\frac{1}{18}=-4x}{-4}$

$\boxed{x=.7226}$

29. $4e^{4x}-18=50$

$4e^{4x}=68$

$e^{4x}=17$

$\frac{ln17=4x}{4}$

$\boxed{x=.7083}$

Applications of Exponentials & Logarithms

Formulas for Compound Interest

For n # of times compounding per year: $A = P(1 + \frac{r}{n})^{nt}$

A = Final Amount   P = Initial Amt Principle   r = interest rate   t = time

annual n = 1   semiannual n = 2   monthly n = 12   quarterly n = 4   daily n = 365

For continuous compounding: $A = Pe^{rt}$

Set up and solve the following problems.

1. You invest $500 in a savings account that pays 3.5% annual interest. How much will be in the account after five years?

$A = 500(1+.035)^5$

$A = \$593.84$

2. You invest $1500 in a savings account that pays 4.5% annual interest compounded semi-annually. How much will be in the account after seven years?

$A = 1500(1 + \frac{.045}{2})^{14}$

$A = \$2048.23$

3. You invest $2000 in a savings account that pays 3.1% annual interest compounded quarterly. How much will be in the account after four years?

$A = 2000(1 + \frac{.031}{4})^{16}$

$A = \$2262.95$

4. You invest $2100 in a savings account that pays 4.2% annual interest compounded monthly. How much will be in the account after three years?

$A = 2100(1 + \frac{.042}{12})^{12.3}$

$A = \$2381.47$

5. You invest $900 in a savings account that pays 5.5% annual interest compounded daily. How much will be in the account after eight years?

$A = 900(1 + \frac{.055}{365})^{365(8)}$

$A = \$1397.39$

6. You invest $1250 in a savings account that pays 6% annual interest compounded continuously. How much will be in the account after ten years?

$A = 1250e^{.06(10)}$

$A = \$2277.65$

3

...invest \$2000 in a savings account that pays 4.7% interest compounded continuously. How long will it ... for the account to have \$3515.78?

$3515.78 = 2000e^{.047t}$ $\quad \frac{\ln(1.75789)}{.047} = \frac{.047t}{.047}$

$1.75789 = e^{.047t}$

$t = 12$ years

8. How long will it take your money to double at 2.4 % compounded continuously?

$200 = 100e^{.024t}$ $\quad \frac{\ln 2}{.024} = \frac{.024t}{.024}$

$2 = e^{.024t}$

$t = 28.88$ years

9. A population of insects is growing in such a way that the number in the population t days from now is given by the formula $P = 4000e^{0.02t}$. How large will the population be in one week? $t = 7$

P = 4601 insects

Given the original principal, the annual interest rate, the amount of time for each investment, and the type of compounded interest, find the amount at the end of the investment.

10. P = \$1000, r = 10%, t = 4 years, monthly

$1000(1+\frac{.10}{12})^{12\cdot 4} = \$1489.35$

11. P = \$3200, r = 6%, t = 5 years 6 months, quarterly

$3200(1+\frac{.06}{4})^{5.5(4)} = \$4440.20$

12. P = \$750, r = 5.5%, t = 3 years 2 months, continuously

$750e^{.055(19/6)} = \$892.69$

13. P = \$45,000, r = 7.2%, t = 30 years, daily

$45000(1+\frac{.072}{365})^{365(30)} = \$390,118.09$

14. How long will it take you to triple your money in an account that earns 3.2% interest compounded monthly?

$300 = 100(1+\frac{.032}{12})^{12t}$ $\quad \frac{\log_{1.0027}(3)}{12} = \frac{12t}{12}$

$3 = (1.0027)^{12t}$

$\boxed{t = 34.38 \text{ years}}$

15. At what interest rate do you need to double you money in 10 years if it is compounded continuously?

$200 = 100e^{r(10)}$

$2 = e^{10r}$ $\quad r = .069315 \rightarrow 6.93\%$ interest rate

$\frac{\ln 2}{10} = \frac{10r}{10}$

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Round all answers to 3 decimal places.

Continuous Growth or decay: $y = y_0 e^{kt}$ where y_0 = initial amount (starting amount)
k = constant of proportionality, t = time
y = ending amount

Half-Life: $y = P(\frac{1}{2})^{h/t}$ or $y = Pe^{rt}$

16. Radium-226, a common isotope of radium, has a half-life of 1620 years. Professor Korbel has a 120 g sample of radium-226 in his laboratory. **Use the store key to keep the value of "k" in the calculator as is.**

a.) Find the constant of proportionality for radium-226.

$60 = 120e^{k(1620)}$

$\frac{1}{2} = e^{k(1620)}$

$\ln\frac{1}{2} = 1620k$

$-.000428 = k$

b.) How many grams of the 120 gram sample will remain after 100 years?

$A = 120e^{k(100)}$

$A = 114.97$ grams

17. The half-life of a radioactive isotope is 9 years.
a) Find the constant "k" for a 20 gram sample.

$10 = 20e^{k(9)}$

$\frac{1}{2} = e^{9k}$

$\frac{\ln\frac{1}{2}}{9} = k$

$k = -.077$

b) How many years will it take for the 20 gram sample to be less than 1 gram?

$1 = 20e^{-.077t}$

$\frac{1}{20} = e^{-.077t}$

$\frac{\ln\frac{1}{20}}{-.077} = \frac{-.077t}{-.077}$

$\boxed{t = 38.897 \text{ years}}$

18. The half-life of a radioactive substance is 2200 years. How long will it take the substance to have a 10% weight loss?

~~900-4500~~ $50 = 100e^{k(2200)}$

$\frac{1}{2} = e^{2200k}$

$\frac{\ln\frac{1}{2}}{2200} = \frac{2200k}{2200}$

$k = -.000315$

$90 = 100e^{kt}$

$\frac{9}{10} = e^{kt}$

$\frac{\ln\frac{9}{10}}{k} = \frac{kt}{k}$

$t = 334.406$ years

19. Bob invested a sum of money in a certificate of deposit (CD) that pays 1.25% interest compounded continuously. If he made the investment on January 1st, 2002, and the account is worth $10,000 on January 1st, 2014, what was the original amount in the account?

$\frac{10{,}000}{e^{.0125(12)}} = \frac{Pe^{.0125(12)}}{e^{.0125(12)}}$

$P = \$8607.08$

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