Name Key

Unit 3 Review

* = No Calculator!

*A radical expression can be written as a rational exponent, and vice versa.

Power / root $\quad \sqrt[3]{x} = x^{1/3} \qquad y^{\frac{4}{5}} = \sqrt[5]{y^4}$

1. *Simplify the following.

a) $\sqrt[3]{x^2yz}$ $\quad \sqrt[3]{x^2yz}$

b) $\sqrt[4]{x^5y^{10}z}$ $\quad xy^2\sqrt[4]{xy^2z}$

c) $\sqrt{75a^{11}b^4c^7}$ $\quad 5a^5b^2c^3\sqrt{3ac}$ (75 = 25 · 3; 11 = 5 + 5 + 1; 7 = 3 + 3 + 1)

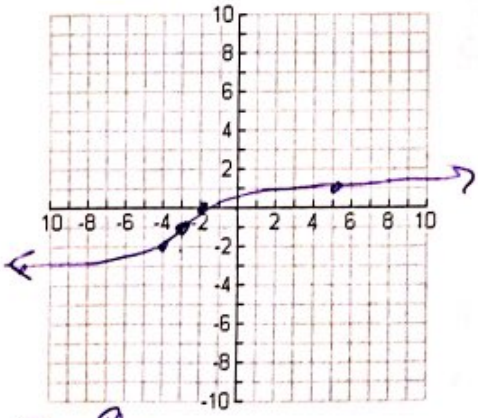
d) $\dfrac{2x^2y^{1/2}}{4x^{-3/4}y^{1/3}}$ $\quad \dfrac{x^{11/4}y^{1/6}}{2}$

e) $(a^4b^6c^{-1/2})^{1/4}$ $\quad \dfrac{ab^{3/2}}{c^{1/8}}$

2. *Graph. State the transformations in order. State the Domain, range, intervals of increasing, decreasing, and end behavior.

a) $y = \sqrt[3]{x+3} - 1$

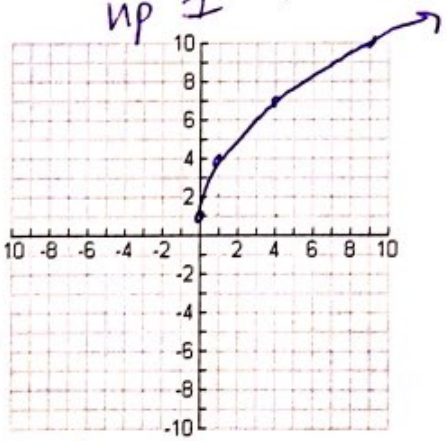
Left 3 ↓1



D: ℝ
R: ℝ
Increases
EB: $x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$

b) $y = 3\sqrt{x} + 1$

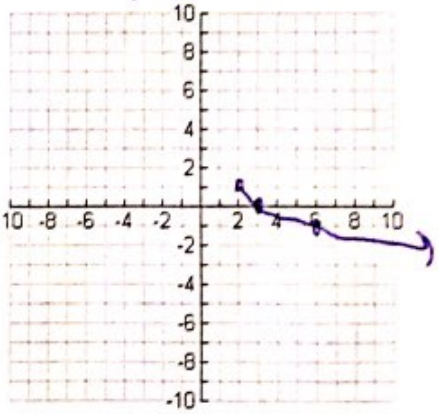
V. stretch by 3
up 1



D: $[0, \infty)$
R: $[1, \infty)$
Increases.
EB: $x \to \infty, y \to \infty$
$x \to -\infty$, y → undef.

c) $y = -\sqrt{x-2} + 1$

Reflect x-axis
R 2 ↑ 1



D: $[2, \infty)$
R: $(-\infty, 1]$
Decreases
EB: $x \to \infty, y \to -\infty$

3. * Working backwards: Writing the equation when given a translation.

a) The parent function $y = 1/x$ is translated 2 units to the left and one unit down.

$$y = \frac{1}{x+2} - 1$$

b) The parent function $y = \sqrt{x}$ is translated 3 units to the right and reflected across the x-axis.

$$y = -\sqrt{x-3}$$

c) The parent function $y = \sqrt{x}$ is compressed vertically by a factor of $\frac{1}{2}$ and 2 units up.

$$y = \frac{1}{2}\sqrt{x} + 2$$

4. Solve each radical equation. **Show all work for full credit!** Be sure to check for extraneous solutions.

a) $\sqrt[3]{x+1} = 4$

$(\sqrt[3]{x+1})^3 = 4^3$

$x+1 = 64$

$\boxed{x = 63}$

b) $5\sqrt{x+7} = 25$

$(\sqrt{x+7})^2 = 5^2$

$x+7 = 25$

$\boxed{x = 18}$

c) $\sqrt{3x-1} = \sqrt{2x+4}$

$(\sqrt{3x-1})^2 = (\sqrt{2x+4})^2$

$3x-1 = 2x+4$

$\boxed{x = 5}$

d) $10 - 2\sqrt{3x-1} = -14$

$-2\sqrt{3x-1} = -24$

$(\sqrt{3x-1})^2 = 12^2$

$3x-1 = 144$

$\boxed{x = \frac{145}{3} \approx 48.33}$

e) $(2x+5)^{\frac{2}{3}} = 16$

$(2x+5)^{\frac{2\cdot 3/2}{3}} = 16^{3/2}$

$2x+5 = \pm 64$

$2x+5 = 64$ $\quad$ $2x+5 = -64$

$2x = 59$ $\quad$ $2x = -69$

$\boxed{x = 29.5}$ $\quad$ $\boxed{x = -34.5}$

f) $(4x+8)^{\frac{1}{2}} = (2x)^2$

$4x+8 = 4x^2$

$0 = 4x^2 - 4x - 8$

$0 = 4(x^2 - x - 2)$

$0 = 4(x-2)(x+1)$

$\boxed{x = 2, -1}$

g) $2 = -x + \sqrt{2x+3}$

$(x+2)^2 = (\sqrt{2x+3})^2$

$x^2 + 4x + 4 = 2x + 3$

$x^2 + 2x + 1 = 0$

$(x+1)(x+1) = 0$

$\boxed{x = -1}$

Direct → $y = kx$ Inverse → $y = \frac{k}{x}$

5. Write an equation of variation to model the following. Then find the missing information.

a. The time to complete a project varies inversely with the number of employees. If 3 people can complete the project in 7 days, how long will it take 5 people?

$3 = \frac{k}{7}$ $y = \frac{21}{x}$ $5 = \frac{21}{x}$

$21 = k$ $x = \frac{21}{5} = 4.2$ days

b. The volume V of a gas kept at a constant temperature varies inversely as the pressure p. If the pressure is 24 pounds per square inch, the volume is 15 cubic feet. What will be the volume when the pressure is 30 pounds per square inch?

$24 = \frac{k}{15}$ $y = \frac{360}{x}$ $30 = \frac{360}{x}$

$360 = k$ $x = 12\ ft^3$

c. The distance a body falls from rest varies directly as the square of the time it falls (ignoring air resistance). If a ball falls 144 feet in three seconds, how far will the ball fall in seven seconds?

$144 = k(3)$ $y = 48x$

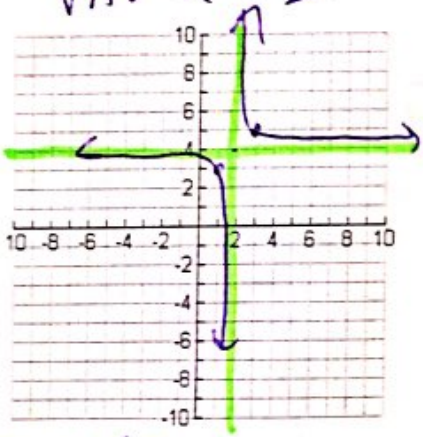
$48 = k$ $y = 48(7) = 336$ ft

6. * Graph the following rational functions. State all asymptotes. Find the domain and range.

a. $y = \frac{1}{x-2} + 4$

HA: $y = 4$

VA: $x = 2$



D: $x \neq 2$

R: $y \neq 4$

Decreases

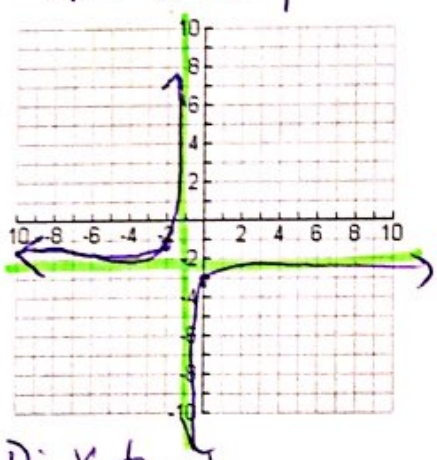
EB: $x \to \infty, y \to 4$

$x \to -\infty, y \to 4$

b. $y = \frac{-1}{x+1} - 2$

HA: $y = -2$

VA: $x = -1$



D: $x \neq -1$

R: $y \neq -2$

Increases

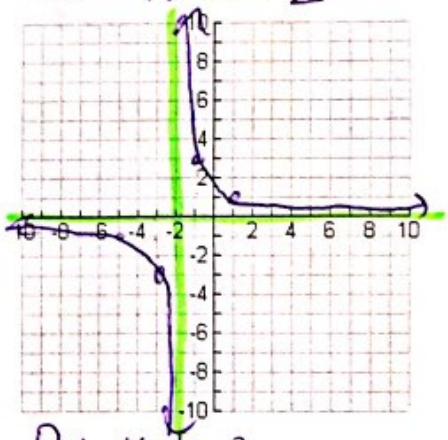
EB: $x \to \infty, y \to -2$

$x \to -\infty, y \to -2$

c. $y = \frac{3}{x+2}$

HA: $y = 0$

VA: $x = -2$



D: $x \neq -2$

R: $y \neq 0$

Decreases

EB: $x \to \infty, y \to 0$

$x \to -\infty, y \to 0$

7. Solve the following rational equations. **Show all work! State any excluded values.**

a. $\frac{1}{3p^2}+\frac{2p-4}{3p^2}=\frac{1^p}{3p}$ EV: $p\neq 0$ CD $=3p^2$

$1+2p-4=p$

$p=3$

b. $\frac{(x-6)^3}{2x}=\frac{1^6}{x}-\frac{1x}{6}$ EV: $x\neq 0$ CD $=6x$

$3x-18=6-x$

$4x=24$

$x=6$

c. $\frac{8}{6x^2+x}=1+\frac{1}{6x^2+x}$ EV: $x\neq 0,\frac{-1}{6}$ CD $=6x^2+x$ $(6x^2+x)$

$8=6x^2+x+1$

$0=6x^2+x-7$

$0=(x-1)(6x+7)$

$x=1,-7/6$

-42, 7, -6, 1

d. $\frac{1}{n+1}=\frac{(n-6)}{n+1}+\frac{n^2+8n+15}{n^2+3n+2}$ $(n+2)$ $(n+2)(n+1)$ CD $=(n+2)(n+1)$ EV: $n\neq -1,-2$

$n+2=n^2-4n-12+n^2+8n+15$

$0=2n^2+3n+1$

$0=(2n+1)(n+1)$

$n=-1,-1/2$

8. Solve the following systems algebraically.

a. $\begin{cases} y=\frac{4}{x}+1 \\ y=2-x \end{cases}$

$2-x=\frac{4}{x}+1$

$x(1-x)=\frac{4}{x}\cdot x$

$x-x^2=4$

$0=x^2-x+4$

$a=1, b=-1, c=4$

$x=\frac{1\pm\sqrt{1^2-4(1)(4)}}{2(1)}$

$=\frac{1\pm\sqrt{-15}}{2}$ → imaginary

No solution

b. y = x + 1 and y = 72/x

$x(x+1)=\frac{72}{x}\cdot x$

$x^2+x=72$

$x^2+x-72=0$

$(x+9)(x-8)=0$

$x=-9, 8$

$(-9,-8)$

$(8,9)$