AFM Unit 1 Day 1 notes

Relations and Functions

Vocabulary

- Relation: a collection ~~set~~ of ordered pairs between 2 sets
- Domain: all x-values; input
- Range : all y-values; output
- Function: For every x, there's only 1 y → no x's repeat
- Vertical Line Test: used on graphs; if passes thru graph more than once, not a function
- Integer: any whole #

Examples

1. Given the relation $\{(-3,2),(1,8),(1,-2)\}$, state the domain and range. Then determine if the relation is a function.

always order least to greatest →

Domain: {-3, 1} Range: {-2, 2, 8} Function? no

2. If x is a negative integer greater than -4, state the relation, D, & R representing the equation $y = x^2 - 5$

Relation: {(-3,4)(-2,-1)(-1,-4)} Domain: {-3,-2,-1} Range: {-4,-1,4}

3. Given $9 < x < 13$, where x is an integer, state the ordered pairs from $y = 8 - 6x$.

Relation: (10,-52) (11,-58) (12,-64)

Is it a function? yes

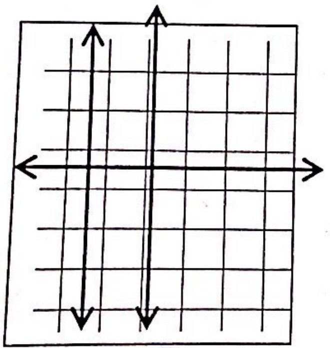
4. Specify one number in the table you could change so that the relation would **NOT** represent a function.

x	1960	1970	1980	1990	2000
y	13,000,000	19,700,000	18,000,000	16,700,000	19,700,000

any x value

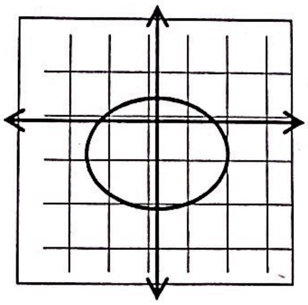
5. Determine if the graph of each relation is the set of a function.

a.



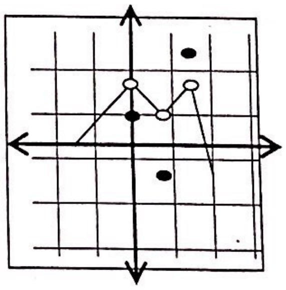
Yes / [✓ No]

b.



Yes / [✓ No]

c.



[✓ Yes] / No

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unction Notation

What is it? f(x), the value of the function, f, at x.
(x, y) or (x, f(x))

6. Find $f(-1)$ if $f(x) = -x^3 - 1$.

$f(-1) = -(-1)^3 - 1$
$= 1 - 1 = 0$
$f(-1) = 0$

7. Find $f(2)$ if $f(x) = 4x - 7$

$f(2) = 4(2) - 7$
$= 8 - 7 = 1$
$f(2) = 1$

8. Find $f(-2)$ if $f(x) = \frac{x-1}{x^2}$.

$f(-2) = \frac{-2-1}{(-2)^2} = \frac{-3}{4}$
$f(-2) = \frac{-3}{4}$

9. Find $f(-3)$ if $f(x) = 2x^2 + 3x - 1$

$f(-3) = 2(-3)^2 + 3(-3) - 1$
$= 18 - 9 - 1 = 8$
$f(-3) = 8$

Review of Interval Notation:

- When a number value IS included, we use Brackets [ ].
- When a number value IS NOT included, we use parenthesis ( )

* ∞ always use ( )!

Graph the following inequalities and write in interval notation.

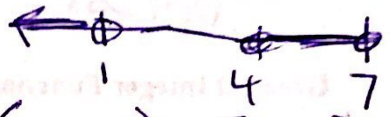
10. $2 < x \leq 5$

(2, 5]

11. $x \geq -3$

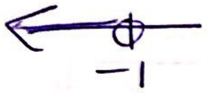
[-3, ∞)

12. $x < 1$ and $4 \leq x \leq 7$



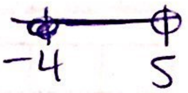
(-∞, 1) ∪ [4, 7]

13. $x < -1$



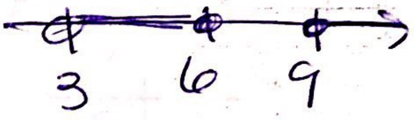
(-∞, -1)

14. $-4 \leq x < 5$



[-4, 5)

15. $3 < x \leq 6$ and $x \geq 9$

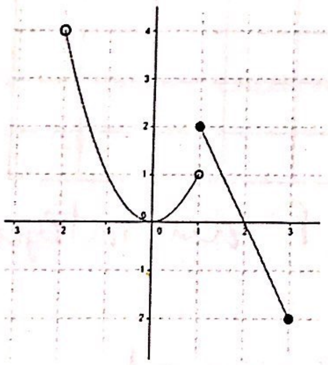


(3, 6] ∪ [9, ∞)

Finding Domain….

***All x values in order from least to greatest! Write it in interval notation.**

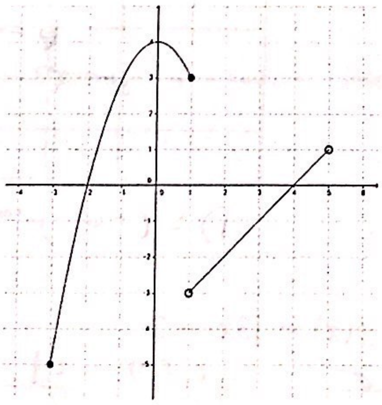
16.



Domain: (-2, 3]

R: [-2, 4)

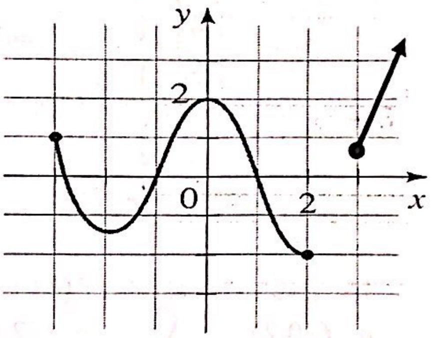
17.



Domain: [-3, 5)

R: [-5, 4]

18.



Domain: [-3, 2] ∪ [3, ∞)

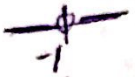
R: [-2, ∞)

Finding Domain Algebraically

- Start with: all ℝs $(-\infty, \infty)$
- Restrictions
  - denominator ≠ zero
  - $\sqrt{\ }$ → # under radical cant be negative

19. State the domain in interval notation.

a. $f(x) = \frac{x^2}{x+1}$ $x+1 \neq 0$ $x \neq -1$

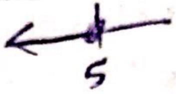


$(-\infty, -1) \cup (-1, \infty)$

b. $f(x) = \sqrt{x+3}$ $x+3 \geq 0$ $x \geq -3$

$[-3, \infty)$

c. $f(x) = \sqrt{5-x}$ $5-x \geq 0$ $5 \geq x$



$(-\infty, 5]$

d. $f(x) = \frac{x}{x^2-81}$ $x^2-81 \neq 0$ $x^2 \neq 81$ $x \neq \pm 9$

$(-\infty, -9) \cup (-9, 9) \cup (9, \infty)$

e. $f(x) = x^2 - 1$

all ℝs or $(-\infty, \infty)$

f. $f(x) = \sqrt{2x-5} + 6$

$2x-5 \geq 0$

$2x \geq 5$

$x \geq 5/2$

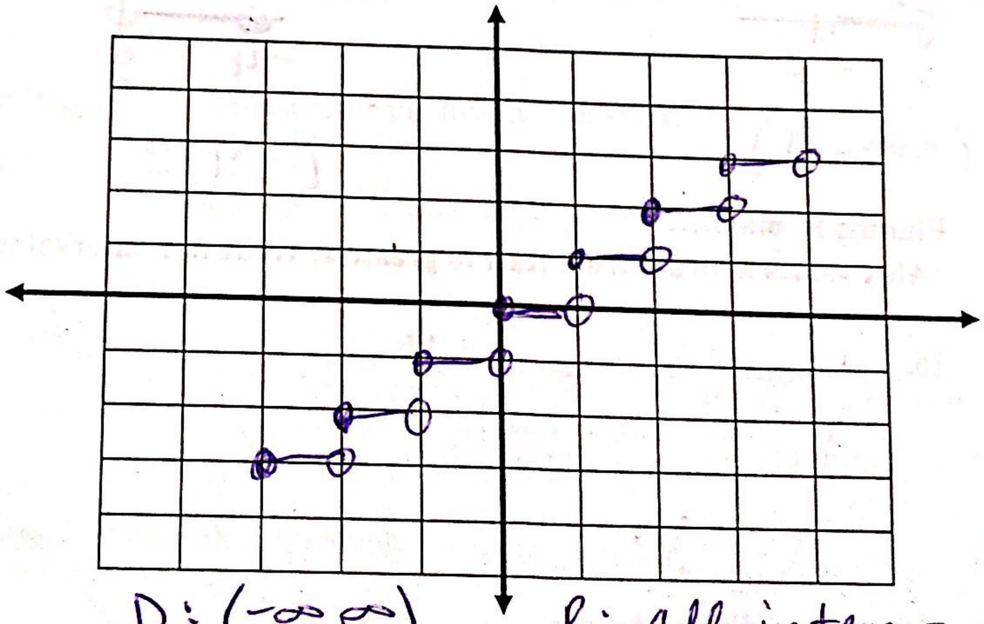
$[5/2, \infty)$

Greatest Integer Function (also known as the birthday function)

A type of step function. The symbol $[x]$ means the integer Not Greater than x. **ROUND DOWN!!!!!**

20. Fill in the table and graph for y = [x]

x	y
-1.7	-2
-1.2	-2
-1.0	-1
-0.9	-1
-0.4	-1
0	0
0.3	0
1.0	1
1.5	1
1.8	1
2.0	2
2.6	2



D: $(-\infty, \infty)$ R: all integers

5. Evaluate each for $f(x) = [x]$ and $g(x) = [3x] - 2$

. $f(0.2) =$ 0 b. $f(2.9) =$ 2 c. $f(9) =$ 9 d. $f(-0.3) =$ -1

$g(-1.2) =$ -6 f. $g(-5.5) =$ -19 g. $g(-6) =$ -20 h. $g(0.1) =$ -2

Transformations

- Reflection: Flip over a line, mirror image
- Translation: slides left, right, up, down
- Stretches/Compressions: multiply → vertical + horizontal

General Form: $y = af[b(x-h)] + k$

*****MUST FACTOR OUT b in order to get h correct**

a: vertical stretch or compression factor

$\frac{1}{b}$: horizontal stretch or compression factor

h: translates left/right

k: translates up/down

Summary of Translations and Transformations

If the equation of $y = f(x)$ is change to:	Then the graph of $y = f(x)$ is:
$y = -f(x)$	Reflect x-axis
$y = f(-x)$	Reflect y-axis
$y = af(x), \lvert a\rvert > 1$	vertical stretch
$y = af(x), 0 < \lvert a\rvert < 1$	vertical compression
$y = f(x-h)$	Left/Right h units (opp sign)
$y = f(x) + k$	↑/↓ k units (same sign)
$y = f(bx), \lvert b\rvert > 1$	horizontal compression
$y = f(bx), 0 < \lvert b\rvert < 1$	horizontal stretch

When listing translations/transformation, **ORDER MATTERS**!!!!!

To describe transformations: it is like order of operations, look at equation from LEFT TO RIGHT Reflections and stretches/compressions first in any order, then translations last in any order.

List the transformations, **in order**, that have occurred when compared to the parent graph.

a. $g(x) = x^2 - 2$

↓2

b. $f(x) = x^2 + 1$

↑1

c. $h(x) = (x-2)^2$

R2

d. $n(x) = (x+3)^2$

L3

e. $m(x) = -(x-3)^2$

① Reflect x-axis

② R3

f. $s(x) = -(x+5)^2$

① Reflect x-axis

② L5

g. $l(x) = (x-2)^2 + 3$

① R2

② ↑3

h. $k(x) = (x+8)^2 - 6$

① L8

② ↓6

i. $a(x) = 3(x+2)^2$

① V. stretch by 3

② L 2

j. $g(x) = \frac{1}{3}(2x+6)^2$

$= \frac{1}{3}(2(x+3))^2$

① V comp $\frac{1}{3}$

② H comp $\frac{1}{2}$

③ L 3

k. $f(x) = (-x+4)^3$

$= (-(x-4))^3$

① Reflect Y-axis

② R 4

l. $h(x) = [\frac{1}{2}(x+2)$ $f(x) = 3x^2 -$

① V

① H. Stretch by 2

② L 2

m. $r(x) = (3x-15)^2$

$= (3(x-5))^2$

① H comp by $1/3$ ② R5

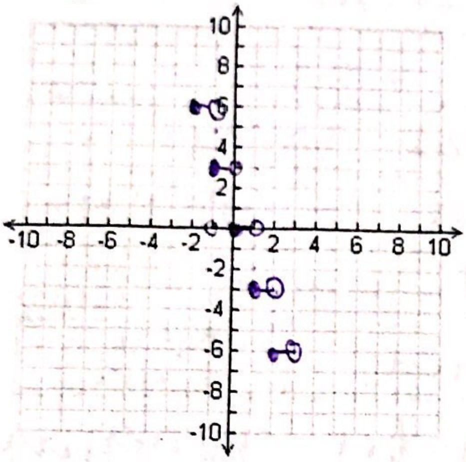
Graph each function. Then describe the transformations.

a. $g(x) = -3[x]$

(-1,-1)	(-1,3)
(0,0)	(0,0)
(2,2)	(2,-6)
(3,3)	(3,-9)

① Reflect x-axis

② V stretch

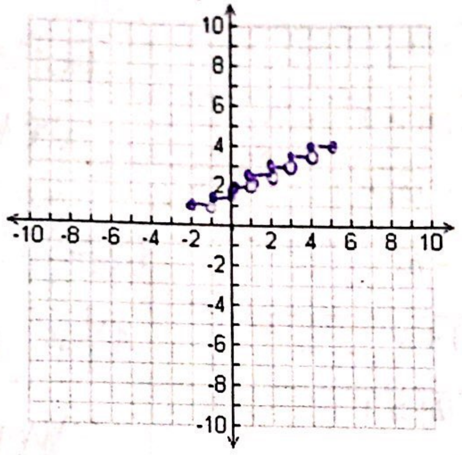


b. $h(x) = 0.5[x] + 2$

(-1,-1)	(-1,$-\frac{1}{2}$)
(0,0)	(0,0)
(1.5,1)	(1.5,$\frac{1}{2}$)
(2,2)	(2,1)
(4,4)	(4,2)

① V comp

② up 2

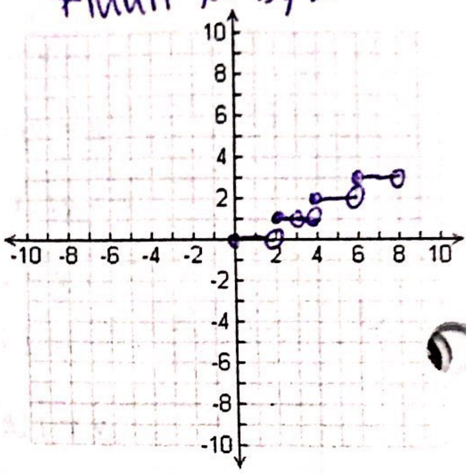


c. $p(x) = [\frac{1}{2}x]$

(-1,-1)	(-2,-
(0,0)	(0,0)
(1,1)	(2,1
(2,2)	(4,2
(3,3)	(6,2

① H. stretch

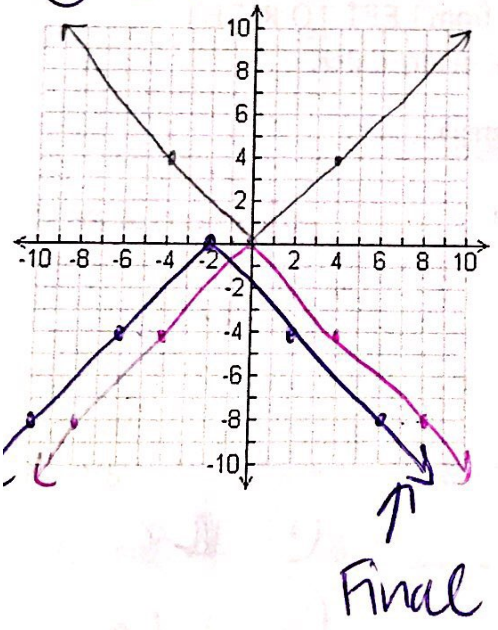
*Mult x's by 2



d. $f(x) = -|x+2|$

① Reflect x-axis

② L 2

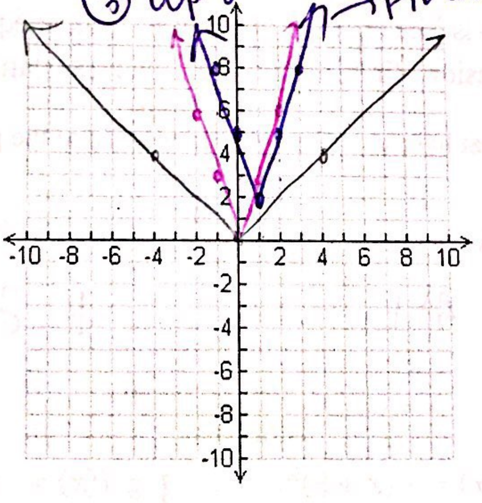


e. $g(x) = 3|x-1| + 2$

① V stretch by 3

② R1

③ up 2

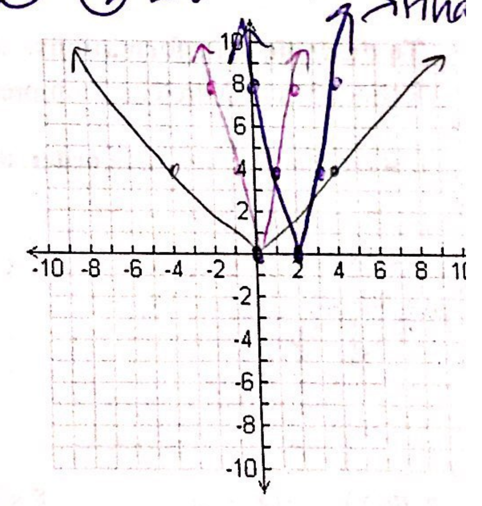


f. $h(x) = |4x+8|$

$|4(x+2)|$

① H comp

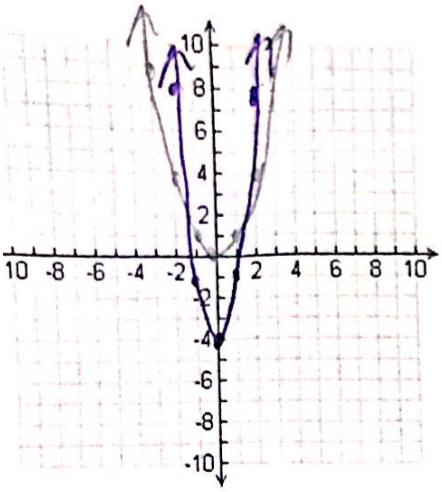
② L 2



$f(x) = 3x^2 - 4$

① V stretch by 3

② ↓ 4



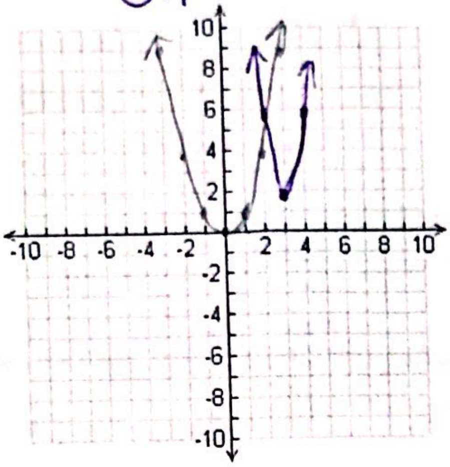
$= (2(x-3))^2 + 2$

h. $g(x) = (2x - 6)^2 + 2$

① H comp $\frac{1}{2}$

② R 3

③ ↑ 2

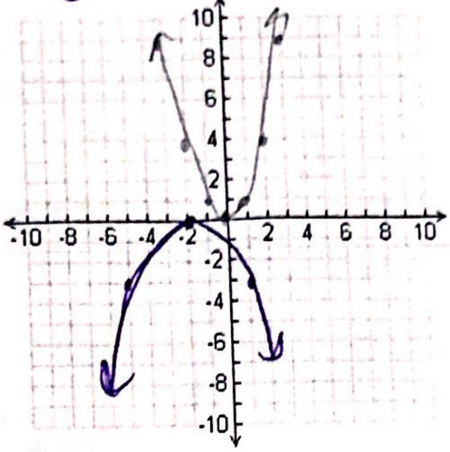


i. $h(x) = -\frac{1}{3}(x + 2)^2$

① Reflect x-axis

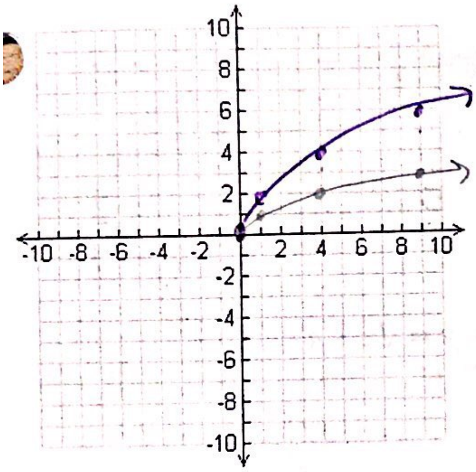
② V comp 1/3

③ L 2



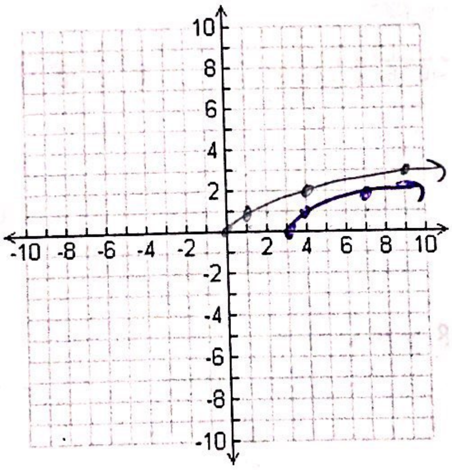
j. $f(x) = 2\sqrt{x}$

V. stretch by 2



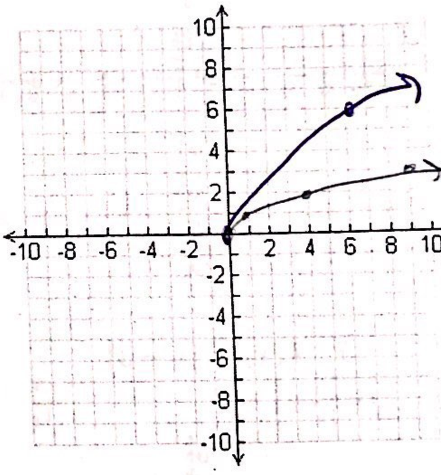
k. $g(x) = \sqrt{x - 3}$

R 3



l. $h(x) = \sqrt{6x}$

H comp by $\frac{1}{6}$



ay 3 Notes - Rational Functions

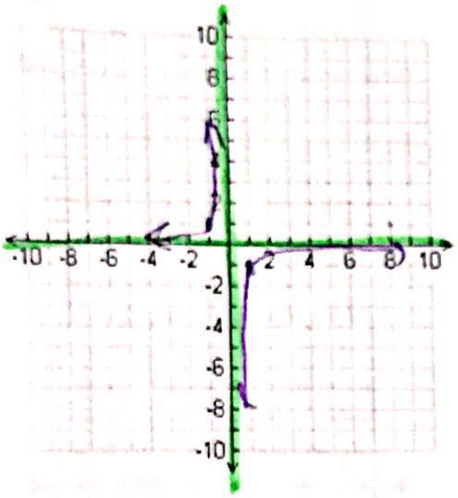
*L/R affects VA

*up/↓ affects HA

1. Describe and Graph each transformation of $y = \frac{1}{x}$.

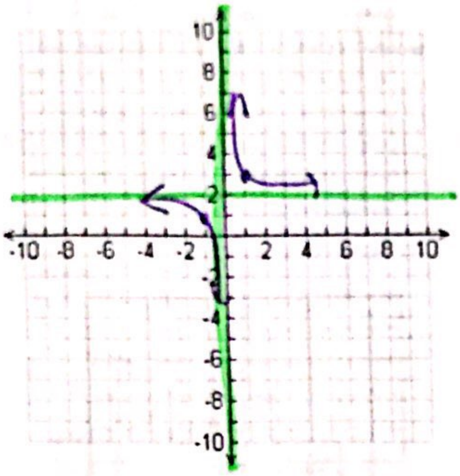
a. $y = -\frac{1}{x}$

Reflect x-axis



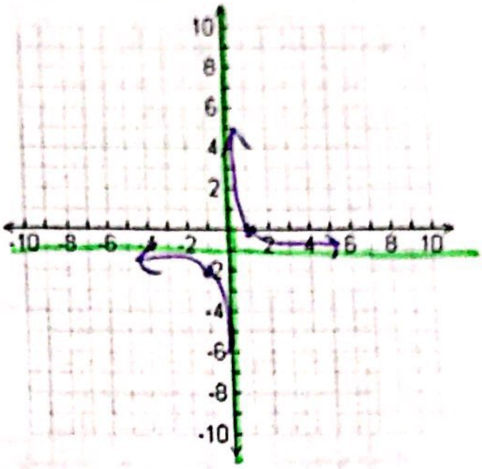
b. $y = \frac{1}{x} + 2$

↑ 2



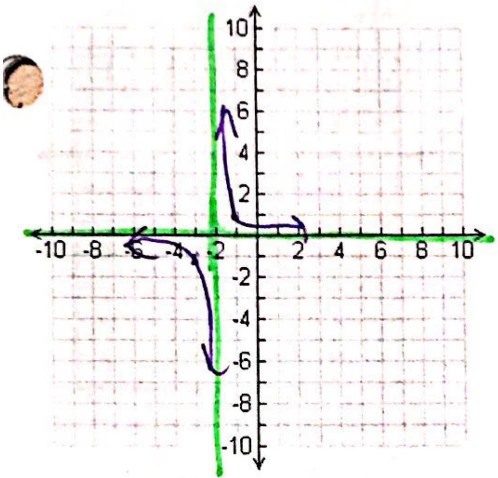
c. $y = \frac{1}{x} - 1$

↓1



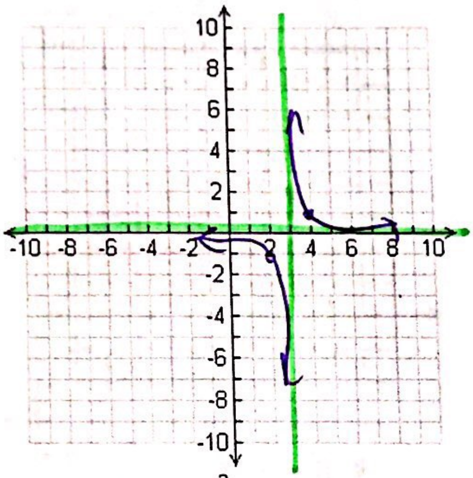
d. $y = \frac{1}{x+2}$

L2



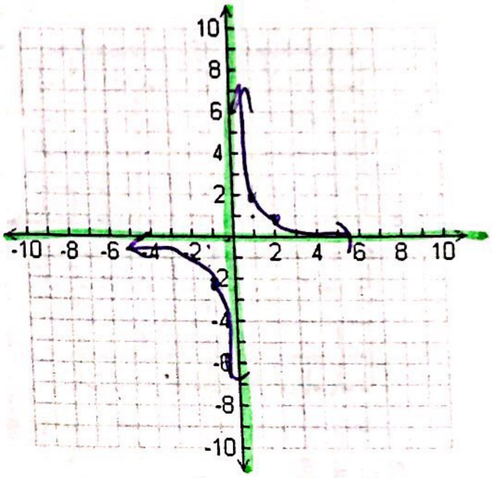
e. $y = \frac{1}{x-3}$

R3



f. $y = \frac{2}{x}$

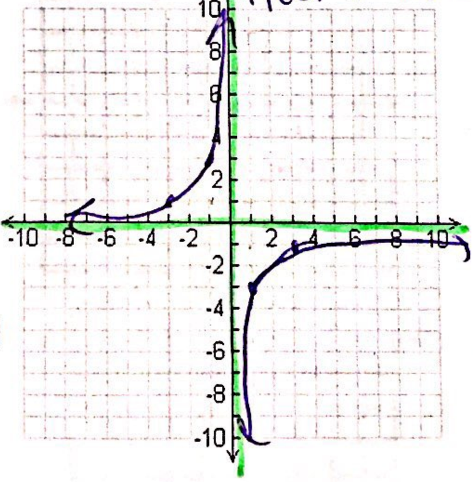
V. stretch by 2



g. $y = -\frac{3}{x}$

-V. stretch 3

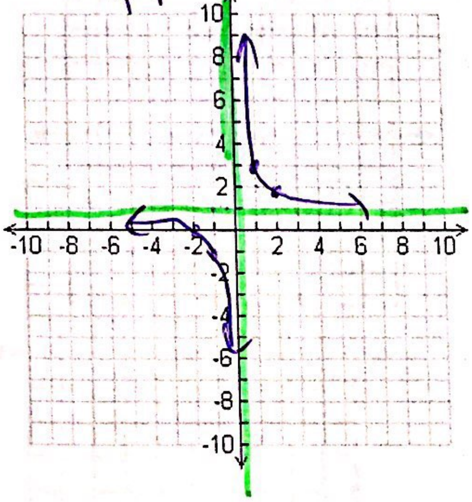
-Reflect x-axis



h. $y = \frac{2}{x} + 1$

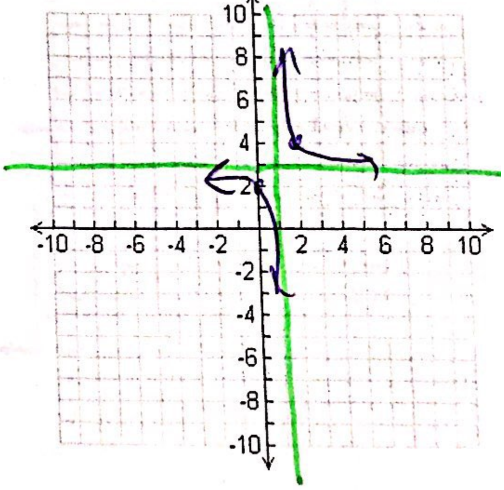
-V. stretch by 2

-↑1



i. $y = \frac{1}{x-1} + 3$

R1, ↑3



Day 4 **Factor Review**

Factor each of the following expressions completely.

a. $x^2 + 7x - 18$

$(x+9)(x-2)$

-48 16/2 -3/2 13

b. $2n^2 + 13n - 24$

$(2n-3)(n+8)$

-60 -12 5/3 3 -4

c. $3m^2 - 7m - 20$

$(3m+5)(m-4)$

-12 1/4 -11

d. $4m^2 - 11m - 3$

$(4m+1)(m-3)$

e. $2w^2 - 8w$

$2w(w-4)$

f. $9x^2 - 25$

$(3x+5)(3x-5)$

Rational Functions Notes **Rational --- FRACTIONAL!!!**

KEY WORDS	HOW TO FIND
Vertical Asymptotes (VA) Dotted vertical lines the graph approaches but never touches (x = #)	FACTOR denominator IF POSSIBLE ✱ Set denominator (factors) = 0, Solve for x
Horizontal Asymptotes (HA) End behavior of the graph Dotted horizontal line Y = # (graph usually does not intersect, but can) At most one horizontal asymptote	HA: $y = \frac{a}{b}$ (*Ratio of LC*) *when* deg *top* = deg *bottom* HA: $y = 0$ *when* deg *top* < deg *bottom* NO Horizontal asymptote: *when* deg *top* > deg *bottom*
Slant Asymptote (SA) Oblique line (neither vertical nor horizontal) If there is NO horizontal asymptote and Degree top > deg bottom by ONE	IF FACTORED LOOK AT ORIGINAL PROBLEM ✱ Use long division or synthetic to find equation of Slant Asymptote & DISREGARD REMAINDER
Hole Point of discontinuity (open circle) When there is a common FACTOR reduced	FACTOR everything and reduce common factors set common factor = 0 to solve for x, plug that value into simplified rational equation to obtain ordered pair (hole) common factor
X intercepts Points on graph that lie on x-axis	✱ set numerator = 0 and solve for x (or let y = 0 and solve for x)
Y intercepts Points on the graph that lie on the y-axis	let x = 0 and solve for y ✱

Examples: Identify the characteristics of each rational function. If the characteristic does not exist, write NONE.

d. $h(x) = -x^2$

1. $y = \frac{3x^2+4}{x+2}$

can't factor

Slant

$$\begin{array}{r|rrr} -2 & 3 & 0 & 4 \\ & \downarrow & -6 & 12 \\ \hline & 3 & -6 & \end{array}$$

VA $\begin{cases} x+2=0 \\ x=-2 \end{cases}$

x-int $\begin{cases} 3x^2+4=0 \\ 3x^2=-4 \\ \sqrt{x^2}=\sqrt{-4/3} \end{cases}$

VA: $x=-2$

HA: ~~y~~ none

SA: $y=3-6x$

Hole: none

X-int(s): none

Y-int: $\frac{4}{2}=2 \rightarrow (0,2)$

2. $y = \frac{2x+1}{3x^2+7x+2}$ $\frac{2x+1}{(3x+1)(x+2)}$

VA $\begin{cases} x+2=0 \\ x=-2 \\ 3x+1=0 \\ x=-1/3 \end{cases}$

$\begin{rcases} 2x+1=0 \\ x=-\frac{1}{2} \end{rcases}$ x-int

VA: $x=-2, -\frac{1}{3}$

HA: $y=0$

SA: none

Hole: none

X-int(s): $(-1/2, 0)$

Y-int: $(0, \frac{1}{2})$

3. Determine the following. Then state the domain.

a. $f(x) = \frac{5x}{x-4}$

VA: $x=4$

HA: $y=5$

SA: none

Hole: none

X-int: $(0,0)$

Y-int: $(0,0)$

Domain:

$(-\infty,4)\cup(4,\infty)$

b. $f(x) = \frac{-2x+1}{3x+5}$

VA: $x=-5/3$

HA: $y=-\frac{2}{3}$

SA: none

Hole: none

X-int: $(\frac{1}{2},0)$

Y-int: $(0, 1/5)$

Domain:

$(-\infty, -\frac{5}{3})\cup(-\frac{5}{3},\infty)$

c. $f(x) = \frac{3}{x^2+4x}$

$x(x+4)$

VA: $x=0, x=-4$

HA: $y=0$

SA: none

Hole: none

X-int: none

Y-int: none

Domain:

$(-\infty,-4)\cup(-4,0)\cup(0,\infty$

d. $h(x) = \frac{x^2-1}{x^2-6x-7}$ $\frac{\cancel{(x+1)}(x-1)}{(x-7)\cancel{(x+1)}}$

VA: $x=7$

HA: $y=1$

SA: none

Hole: $(-1, \frac{1}{4})$ $\frac{-1-1}{-1-7} = \frac{-2}{-8} = \frac{1}{4}$

X-int: $(1,0)$

Y-int: $(0, 1/7)$

Domain: $(-\infty, -1) \cup (-1, 7) \cup (7, \infty)$

e. $g(x) = \frac{x^2-6x+9}{x^2-x-6}$

$\frac{\cancel{(x-3)}(x-3)}{\cancel{(x-3)}(x+2)}$

VA: $x=-2$

HA: $y=1$

SA: none

Hole: $(3, 0)$ $\frac{3-3}{3+2} = \frac{0}{5}$

X-int: none

Y-int: $(0, -3/2)$

Domain: $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

f. $m(x) = \frac{x^2+6x+8}{x+4}$

$\frac{\cancel{(x+4)}(x+2)}{\cancel{x+4}}$

VA: none

HA: none

SA: $y=x+2$

Hole: $(-4, -2)$ $-4+2=-$

X-int: $(-2,0)$

Y-int: $(0,2)$

Domain: $(-\infty, -4) \cup (-4, \infty$

g. $f(x) = \frac{12x}{3x^2+1}$

VA: none

HA: $y=0$

SA: none

Hole: none

X-int: $(0,0)$

Y-int: $(0,0)$

Domain:

$(-\infty, \infty)$

h. $f(x) = \frac{2x^3}{x^2+1}$

$$x^2+1 \overset{2x}{\overline{\big) 2x^3}}$$

$$-(2x^3+2x)$$

VA: none

HA: none

SA: $y=2x$

Hole: none

X-int: $(0,0)$

Y-int: $(0,0)$

Domain:

$(-\infty, \infty)$

i. $f(x) = \frac{x^2-9}{x+2}$

$$\begin{array}{r|rrr} -2 & 1 & 0 & -9 \\ & \downarrow & -2 & 4 \\ \hline & 1 & -2 & \end{array}$$

VA: $x=-2$

HA: none

SA: $y=x-2$

Hole: none

X-int: $(3,0)(-3,0)$

Y-int: $(0, -9/2)$

Domain:

$(-\infty, -2) \cup (-2, \infty)$

Day 5 - Rational Functions Part 2

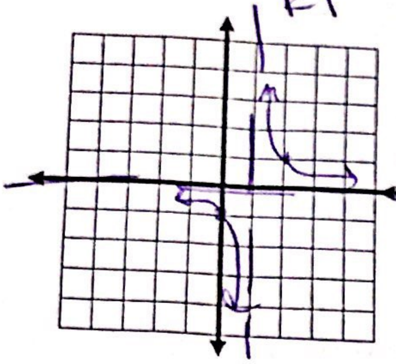
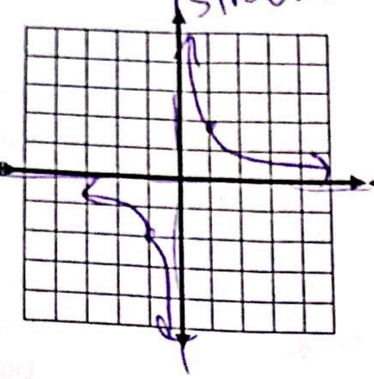
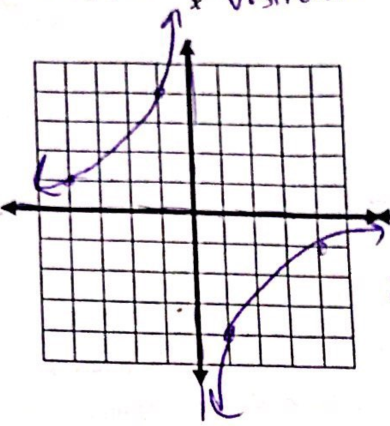
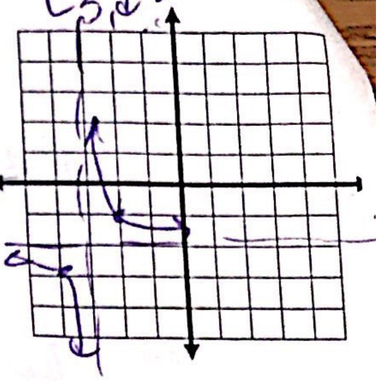
1. Use the parent graph $f(x) = \frac{1}{x}$ to graph each equation. Describe transformation(s) that have taken place. Then list the domain and range.

a. $g(x) = \frac{1}{x-1}$ R1

b. $f(x) = \frac{2}{x}$ V stretch

c. $k(x) = -\frac{4}{x}$ Reflect V.stretch

d. $y = \frac{1}{x+3} - 2$ L3, d2

2. Write an equation of a function that has a vertical asymptote at $x = -1$ and a hole at $x = 4$ $\frac{(x-4)}{(x+1)(x-4)}$

3. Write an equation of a function that has a slant asymptote. $y = \frac{x^3}{x^2}$

4. When graphing a rational function that is not a standard transformation of $\frac{1}{x}$ then there are some steps to take to make it easier to graph, **without** using a graphing calculator.

   a. Factor and simplify if possible.
   b. Identify any vertical, horizontal, slant asymptotes and holes.
   c. Find the y-intercept, if one
   d. Find the x-intercepts, if any.
   e. Start sketch with the information you have

5. Graph each function.

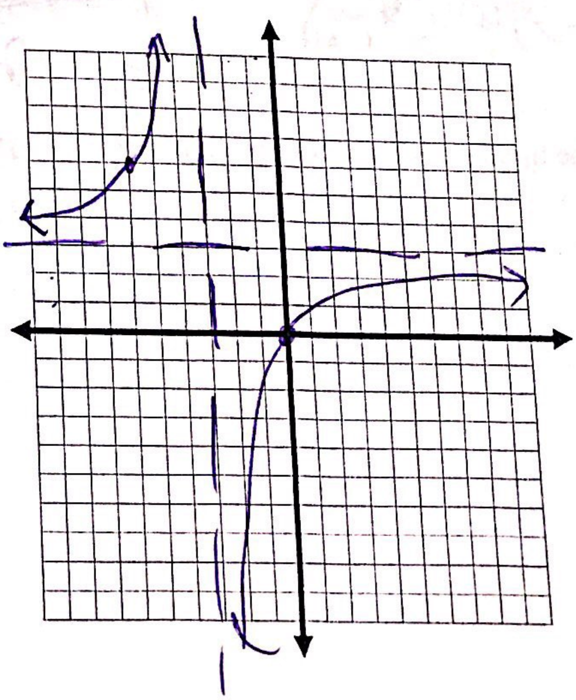
a. $f(x) = \frac{3x}{x+3}$

VA: $x = -3$

HA: $y = 3$

X-int: $(0,0)$

y-int: $(0,0)$



b. $f(x)=\dfrac{x}{2x^2-x-10}$

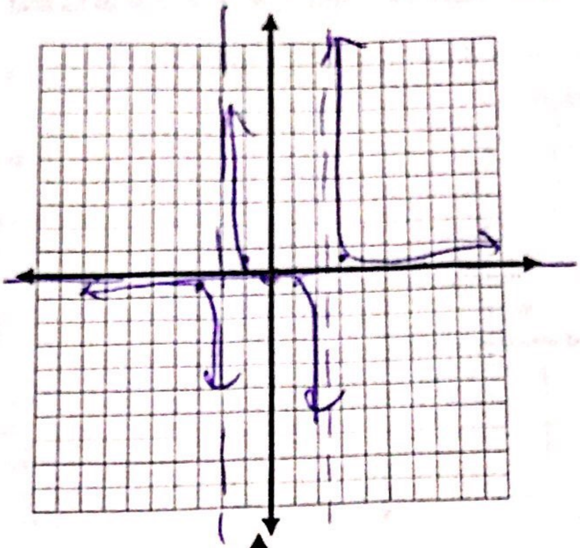
$(2x-5)(x+2)$

VA: $x=\frac{5}{2}, x=-2$

HA: $y=0$

x-int: $(0,0)$

y-int: $(0,0)$



c. $h(x)=\dfrac{2x^2-3x-2}{x-2}$

$\dfrac{(2x+1)\cancel{(x-2)}}{\cancel{x-2}}$

VA: none

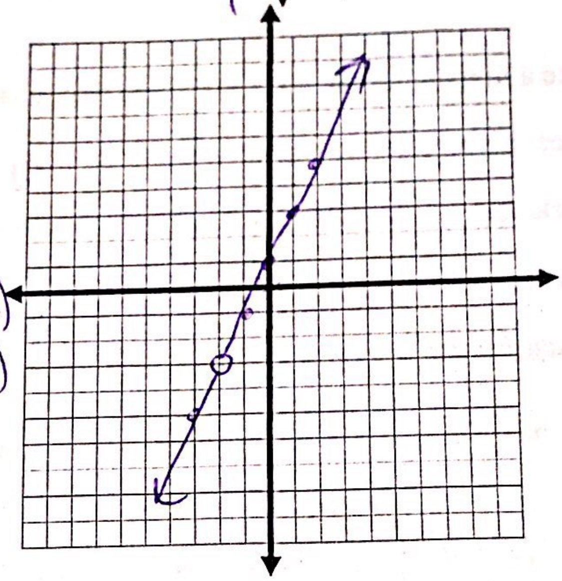
Hole: $(-2,-3)$

HA: none

SA: $y=2x+1$

x-int: $(0,-\frac{1}{2})$

y-int: $(0,1)$



d. $f(x)=\dfrac{x^2+4x-5}{x+1}$

$\dfrac{(x+5)(x-1)}{x+1}$

$$\begin{array}{r|rrr} -1 & 1 & 4 & -5 \\ & \downarrow & -1 & \\ \hline & 1 & 3 & \end{array}$$

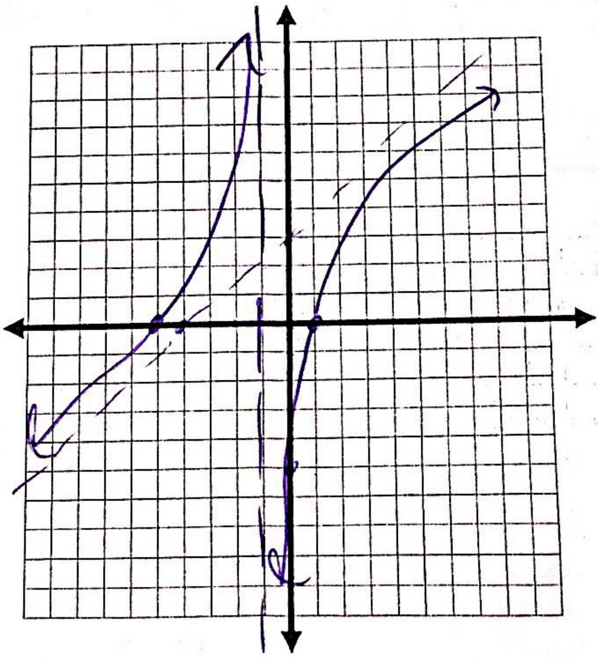
VA: $x=-1$

HA: none

SA: $y=x+3$

x-int: $(0,-5)(0,1)$

y-int: $(0,-5)$



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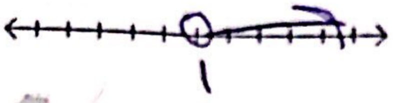
Day 6 Notes Piecewise Functions

1. Piecewise Functions-A functions that is defined by two (or more) equations over a specified domain.

2. Graph each of the following on a number line.

$-3 \geq x$

a. $x > 1$



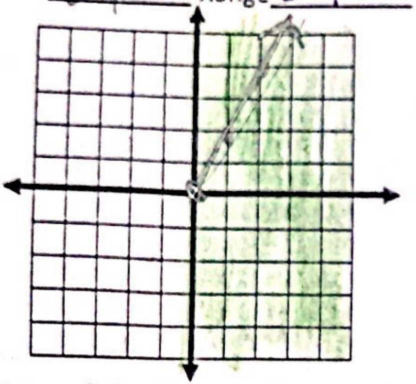
b. $-2 \leq x < 2$

c. $x \leq -3$

3. Graph each of the following, given the domain restrictions. List the domain and range in interval notation.

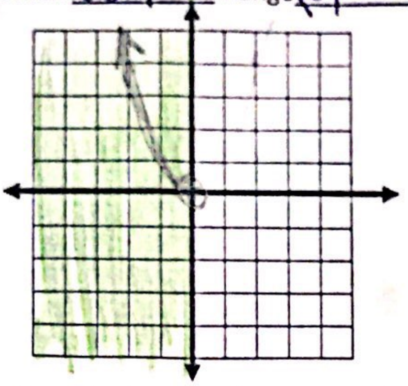
a. $y = 2x, x \geq 0$

Domain $[0,\infty)$ Range $[0,\infty)$



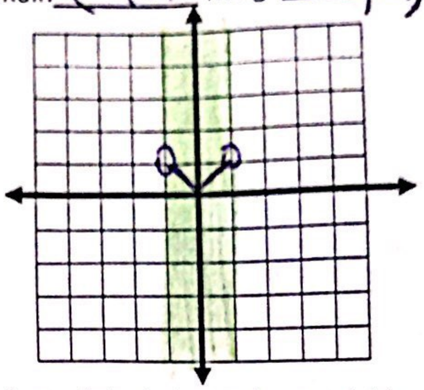
b. $y = x^2, x < 0$

Domain $(-\infty,0)$ Range $(0,\infty)$

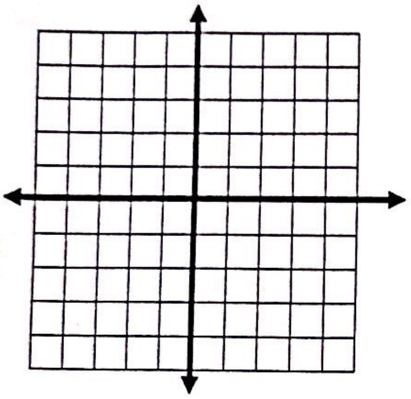


c. $y = |x|, -1 < x < 1$

Domain $(-1,1)$ Range $[0,1)$



d. $y = [x], -2 \leq x \leq 2$



4. Evaluate each piecewise functions at the given values of the independent variable.

a. $g(x) = \begin{cases} 3x + 5 \text{ if } x < 0 \\ 4x + 7 \text{ if } x \geq 0 \end{cases}$

g(-2)
$3(-2)+5$
$-6+5=\boxed{-1}$

g(0) 7

g(3) 19

b. $f(x) = \begin{cases} x + 3 \text{ if } x \geq -3 \\ -(x+3) \text{ if } x < -3 \end{cases}$

f(0)
$0+3=\boxed{3}$

f(-6) 3
$-(-6+3)$
$=-(-3)$

f(-3) 0

c. $h(x) = \begin{cases} \frac{x^2-9}{x-3} \text{ if } x \neq 3 \\ 6 \text{ if } x = 3 \end{cases}$

h(5)
$\frac{\overset{25}{5^2}-9}{5-3} = \frac{16}{2} = \boxed{8}$

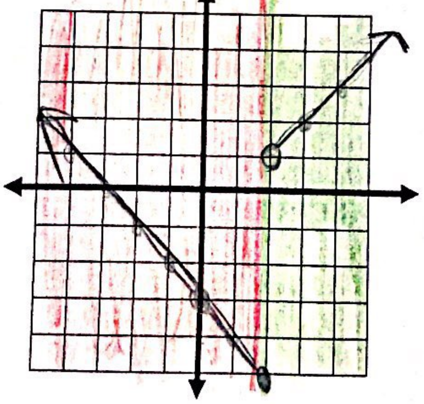
h(0) 3

h(3) 6

5. Sketch the graph of each function. Then list the domain and range in interval notation.

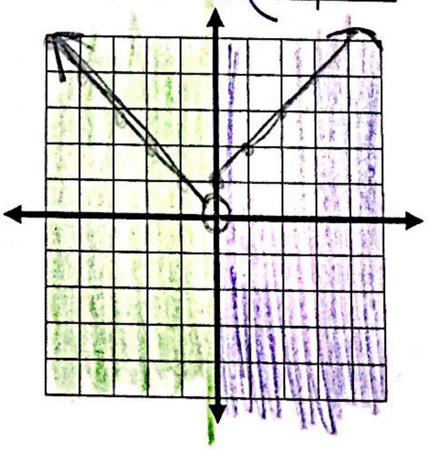
a. $h(x) = \begin{cases} x - 1, x > 2 \\ -x - 3, x \leq 2 \end{cases}$

Domain $(-\infty,\infty)$ Range $[-5,\infty)$



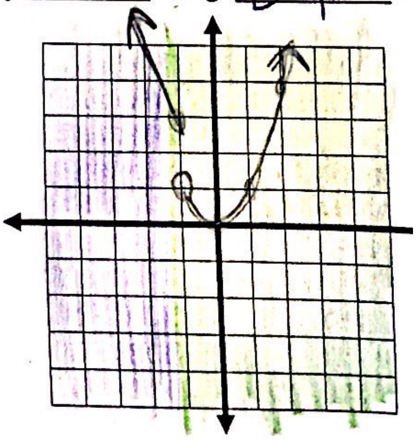
b. $g(x) = \begin{cases} -x, \text{if } x < 0 \\ x + 1, \text{if } x \geq 0 \end{cases}$

Domain $(-\infty,\infty)$ Range $(0,\infty)$



c. $h(x) = \begin{cases} x^2, x > -1 \\ -3x, x \leq -1 \end{cases}$

Domain $(-\infty,\infty)$ Range $[0,\infty)$



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AFM Piecewise Applications

1. An amusement park charges $100 for groups of 10 or less people. For groups of more than 10 they charge the $100 fee plus an additional $4 per person. The park does not allow groups larger than 40.

a. A group of 15 would pay: 160 $100+20=120$

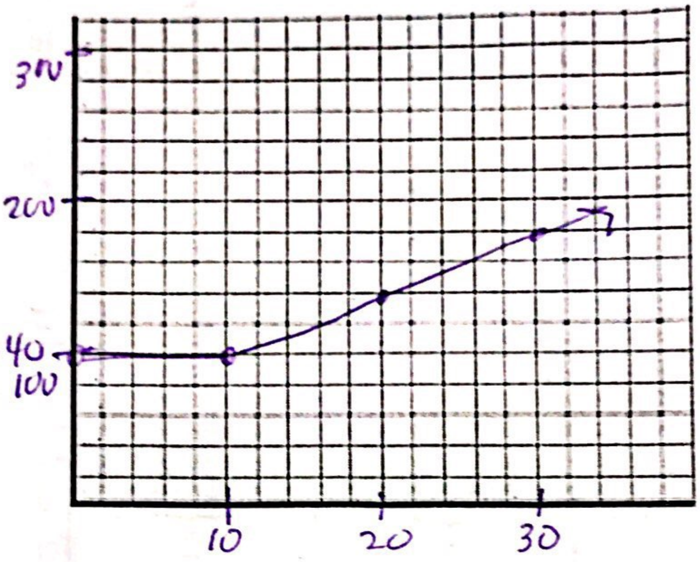
b. A group of 20 would pay: 180 $100+40=140$

Let x = # kids

c. Write a function C = f(x), that represents the cost as a function of the number of people going to the amusement park.

Graph the function.
Label axes.

$$f(x)=\begin{cases}100 & 0<x\le 10\\ 100+4(x-10) & 10<x\le 40\end{cases}$$

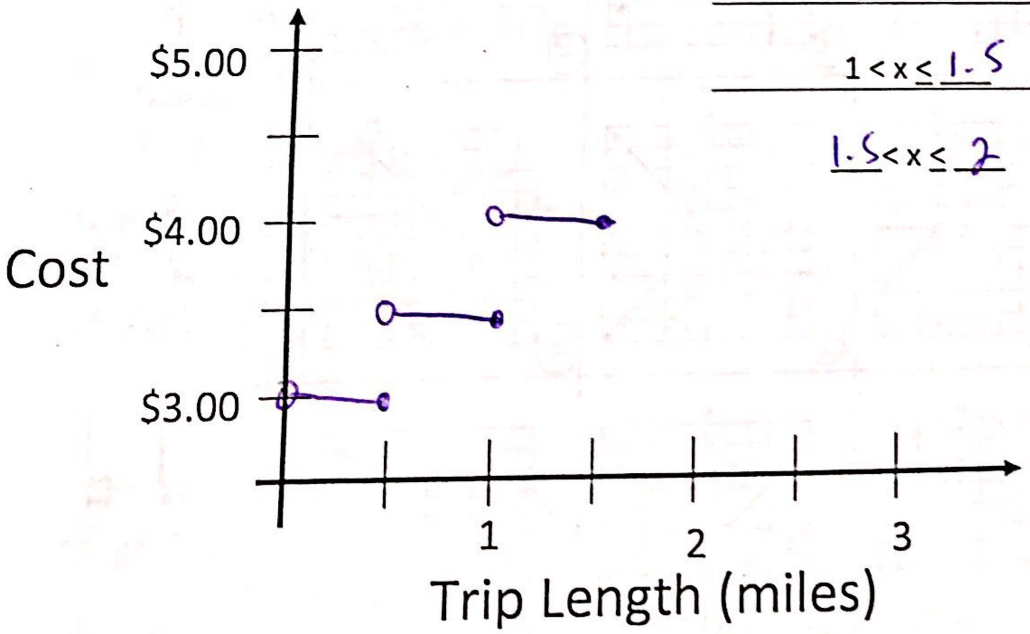


2. A taxi in Los Angeles costs $3.00 for the first half mile and then $0.50 for each additional half mile. The taxis round up to the next half-mile. (So someone on a 1.2 mile ride would be charged for 1.5 mile: $3.00 + 2 (.50) = $4.00)

a) Finish the table

Length of Ride	Cost
$0 < x \le ½$	3.00
$\frac{1}{2} < x \le 1$	$3.50
$1 < x \le 1.5$	4.00
$1.5 < x \le 2$	$4.50

b) Graph the function:



charge

. A long distance telephone charges 99 cents for any call up to 20 minutes in length and 7 cents for each additional minute.

$x = \#$ minutes

a. Use bracket notation to write a formula for the cost, C, of a call as a function of its length time, t, in minutes.

$$C = \begin{cases} .99 & 0 < x \leq 20 \\ .99 + .07(x-20) & 20 < x \end{cases}$$

b. How much does it cost to talk for 10 minutes? .99 25 minutes? 1.34

4. Your favorite dog groomer charges according to your dog's weight. If your dog is under 15 pounds, the groomer charges $35. If your dog is between 15 and 40 pounds, she charges $40. If your dog is over 40 pounds, she charges $40 plus and additional $2 for each extra pound.

a. Find a function that models this situation. $x = \#$ lbs

$$f(x) = \begin{cases} 35 & 0 \leq x < 15 \\ 40 & 15 \leq x \leq 40 \\ 40 + 2(x-40) & x > 40 \end{cases}$$

What would the groomer charge if your cute dog weighs 60 pounds?

$80

5. You have a summer job that pays double time for overtime. That means if you work more than 40 hours in a week, you get paid twice your normal hourly wage of $7.25.

$x = \#$ hrs

a. Write a function that represents how much you make in a week given any number of hours you work.

$$f(x) = \begin{cases} 7.25x & 0 \leq x \leq 40 \\ 290 + 14.50(x-40) & x > 40 \end{cases}$$

b. How much money will you make in a week if you work 48 hours?

$406

6. The zoo charges $15 per person for groups of fewer than 50 people. Groups of 50 people or more are charged a reduced rate of $10 per person.

$x = \#$ ppl

a. Write a function to model this situation.

$$f(x) = \begin{cases} 15x & 0 < x < 50 \\ 10x & x > 50 \end{cases}$$

b. If you have a group of 57 people, how much will it cost to go to the zoo?

$ 570