Unit 5 Sequences & Series Study Guide

Name key

Set up each problem and solve. Show all work!

1. Which term of the arithmetic sequence 18, 11, 4, … is −73?

$d = -7$ $\quad -73 = 18 + (n-1)(-7)$

$-91 = (n-1)(-7)$ $\quad \boxed{n = 14}$

2. Find the sequence with three arithmetic means between 55 and 115.

55, 70, 85, 100, 115 $\quad 115 = 55 + (5-1)d$

$15 = d$

3. Find the eleventh term of the arithmetic sequence $8, \frac{11}{2}, 3, \ldots$

$d = -2.5$ $\quad a_n = 10.5 - 2.5n$ $\quad \boxed{a_{11} = -17}$

4. Find the sum of the first 17 terms in the arithmetic series $3 + 8 + 13 + \ldots$

$d = 5$ $\quad a_{17} = -2 + 5n$ $\quad a_{17} = 83$

$S = \frac{17}{2}(3 + 83) = \boxed{731}$

5. Write $7 + 10 + 13 + \ldots + 31$ using sigma notation.

$d = 3$ $\quad 31 = 4 + 3n$ $\quad 9 = n$

$\sum_{n=1}^{9} (4 + 3n)$

6. Given the sequence 21, 13, 5, …, write the recursive and explicit formula. Then find the 50th term.

E: $a_n = 29 - 8n$ $\quad$ R: $a_n = a_{n-1} - 8$, $a_1 = 21$

$a_{50} = -371$

7. Given the sequence -2, 6, -18, …, write the recursive and explicit formula. Then find the 10th term.

$r = -3$ $\quad$ E: $a_n = -2(-3)^{n-1}$ $\quad$ R: $a_n = -3a_{n-1}$, $a_1 = -2$

$a_{10} = 39366$

8. Find $\sum_{n=1}^{40}(2n + 5)$

$S = \frac{40}{2}(7 + 85) = \boxed{1840}$

9. Find $\sum_{n=1}^{6}(4 - n^2)$

$3 + 0 - 5, -12 - 21 - 32 = \boxed{-67}$

10. Find the sum of the arithmetic series with $a_1 = 70, a_n = 7, d = -3$

$7 = 73 - 3n$

$22 = n$

$S = \frac{22}{2}(70 + 7) = \boxed{847}$

11. Find the fifth term of the geometric sequence 54, -36, 24, …

$r = \frac{-36}{54} = -2/3$ $\quad a_5 = 54(-2/3)^{5-1} = \boxed{\frac{32}{3} = 10.67}$

12. Find the sequence with three geometric means between 24 and $\frac{3}{2}$.

24, 12, 6, 3, 3/2 $\quad \frac{3}{2} = (24)(r)^{5-1}$

$\sqrt[4]{\frac{1}{16}} = \sqrt{r^4}$

$r = 1/2$

13. Which term of the geometric sequence 10, -50, 250, … is 6250?

$r = -5$ $\quad 6250 = 10(-5)^{n-1}$

$625 = -5^{n-1}$

$1 + \frac{\log 625}{\log 5} = n - 1 \log 5$

$\boxed{n = 5}$

14. Evaluate $\sum_{k=1}^{5}(-2)^{k-1}$

$S = \frac{1(1 - -2^5)}{1 - -2} = \boxed{11}$

15. Find the sum of the geometric series $a_1 = 48, r = \frac{1}{2}$, and $n = 5$.

$S = \frac{48(1 - \frac{1}{2}^5)}{1 - 1/2} = \boxed{93}$

16. In the geometric series, given $n = 8, r = 2, S_n = 765$, find a_1 and a_8.

$765 = \frac{a_1(1 - 2^8)}{1 - 2}$ $\quad -765 = a_1(-255)$ $\quad \boxed{3 = a_1}$

$a_8 = 3(2)^{8-1}$ $\quad \boxed{a_8 = 384}$

17. Find the sum of the infinite geometric series $6 - 3 + \frac{3}{2}, \ldots$

$r = -1/2$

$S = \frac{6}{1 - -\frac{1}{2}} = \boxed{4}$

18. Expand $(2x - y^2)^4$ using Pascal.

$(2x)^4 + 4(2x)^3(-y^2) + 6(2x)^2(-y^2)^2 + 4(2x)(-y^2)^3 + (-y^2)^4$

$\boxed{= 16x^4 - 4x^3y^2 + 24x^2y^4 - 8xy^6 + y^8}$

19. Expand $(a+b)^5$

$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

20. Find the fourth term in the expansion $(2x-y)^8$

$56(2x)^5(-y)^3 = \boxed{-1792x^5y^3}$

21. Find the term in the expansion of $(3x+2y)^5$ containing y^3.

$10(3x)^2(2y)^3$

$= \boxed{720x^2y^3}$

22. Expand $(a-2b)^4$ using binomial theorem.

$a^4 + 4(a)^3(-2b) + 6(a)^2(-2b)^2 + 4(a)(-2b)^3 + (-2b)^4$

$\boxed{= a^4 - 8a^3b + 24a^2b^2 - 32ab^3 + 16b^4}$

23. Find the sum of the infinite series $\frac{1}{2} + \frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \ldots$ $r = 2/3$

$S = \frac{\frac{1}{2}}{1 - 2/3} = \boxed{1.5}$

24. Write the first four terms in the sequence formed by the pattern $a_n = 3n - 2$.

1, 4, 7, 10

25. Find the limit $\lim_{n\to\infty} \frac{2n+1}{3n+1}$

2/3

26. Find the limit $\lim_{n\to\infty} \frac{8n^2 - 3n}{5n^2 + 7}$

8/5

27. Find the limit $\lim_{n\to\infty} \frac{2n+1}{3}$

none

29. Given the sequence $t_1 = 3, t_n = 2t_{n-1} + 1$, list the first five terms in the sequence.

3, 7, 15, 31, 63

30. Nicole starts a college savings account for her daughter on her sixth birthday. She plans to deposit $25 the first month and then increase the deposit by $5 each month. How much will she have deposited in twelve years?

A

$d = 5$

$S = \frac{144}{2}(25 + 740) = \boxed{\$55,080}$

144 months

$a_n = 20 + 5n$

$a_{144} = 740$