AFM Unit 6 - Probability Study Guide

Name Key

Formulas to remember:

Permutations:

P(n, n): $n!$

P(n, r): $\frac{n!}{(n-r)!}$

Combinations:

C(n, r): $\frac{n!}{(n-r)!\,r!}$

Geometric Probability:

$\frac{\text{Area of want}}{\text{Area of total}}$

~~Not Fixed:~~

~~Fixed Point:~~

P(A and B):

$P(A) \cdot P(B)$

P(A or B):

$P(A) + P(B) - P(\text{Both})$

Binomial Probability:

$${}_nC_r \cdot p^r \cdot (1-p)^{n-r}$$

Find the following. First decide if it is a permutation or a combination!

1. How many subcommittees of 2 Democrats and 3 Republicans can be formed from a committee whose membership is 6 Democrats and 8 Republicans?

$${}_6C_2 \cdot {}_8C_3 = \frac{6!}{4!\,2!} \cdot \frac{8!}{5!\,3!} = 840$$

2. In how many ways can 5 people be seated in a room containing 2 chairs?

$${}_5P_2 = \frac{5!}{3!} = 20$$

3. A quality control engineer must inspect a sample of 3 fuses from a box of 100. How many different samples can he choose?

$${}_{100}C_3 = \frac{100!}{97!\,3!} = 161{,}700$$

4. How many three-digit numbers can be formed from the digits 2, 3, 4, 5, and 6? Assume there is no repetition of digits.

$${}_5P_3 = \frac{5!}{2!} = 60$$

5. How many 3 letter and 4 digit license plates are there if you can repeat numbers but not letters?

$$\underset{L}{26} \cdot \underset{L}{26} \cdot \underset{L}{26} \cdot \underset{\#}{10} \cdot \underset{\#}{10} \cdot \underset{\#}{10} \cdot \underset{\#}{10} = 15{,}600{,}000$$

6. A witness to a holdup reports that the license of the getaway car consisted of 6 different digits. He remembers the first three but has forgotten the rest. How many licenses do the police have to check?

$1 \quad 1 \quad 1 \quad 10 \quad 10 \quad 10 = 1000$

7. In how many ways can the letters from the word ***television*** be arranged?

$\frac{10!}{2!2!} = 907,200$ (2 Es, 2 Is)

8. How many ways can 8 people be seated at a round table?

OMIT

9. A clown has 8 balloons, each a different color. There are 6 children. How many ways can the clown give each child a balloon?

$_8P_6 = \frac{8!}{2!} = 20160$

10. How many 9-member baseball teams can be formed from 15 players if only 3 pitch while the others play the remaining 8 positions? (You only want 1 pitcher).

$_3C_1 \cdot {}_{12}C_8 = \frac{3!}{2!} \cdot \frac{12!}{4!8!} = 1485$ (Pitcher)

11. A photographer is taking a picture of a bride and a groom together with 4 attendants. How many ways can he arrange the 6 people in a line if the bride and groom stand in the middle?

$4 \quad 3 \quad \underset{B/G}{2} \quad \underset{B/G}{1} \quad 2 \quad 1 = 48$

12. You are buying a new car. You can either pick a Honda Civic or an Accord. You can then choose from red, black, gray, white, or blue for the exterior color. The interior can be tan or light gray and then you can pick from either leather or cloth. How many options for a new car do you have?

$2(5)(2)(2) = 40$

Find the following probabilities. Show all work.

13. Two dice are rolled. What is the probability that their sum is 6 or 8?

$\frac{5}{36} + \frac{5}{36} = \frac{5}{18}$

14. A bag contains 4 red balls and 2 white balls. If two different balls are selected are random (without replacement), what is the probability of getting:

(a) both red

$\frac{4}{6} \cdot \frac{3}{5} = \frac{2}{5}$

(b) one of each color? RW WR

$2\left(\frac{4}{6} \cdot \frac{2}{5}\right) = \frac{8}{15}$

15. Find the probability of drawing two aces from a deck of cards if the first card is not replaced before the second is drawn. $P(A,A) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$

16. A box contains 10 red, 8 green, and 12 blue tickets. Two successive tickets are drawn without replacement. Find the probability of drawing (without regard to order): 30 total

a) one blue and one green ticket

$2\left(\frac{12}{30} \cdot \frac{8}{29}\right) = \frac{32}{145}$

BG, GB

b) two red tickets

$\frac{10}{30} \cdot \frac{9}{29} = \frac{3}{29}$

c)no blue ticket

$\frac{18}{30} \cdot \frac{17}{29} = \frac{51}{145}$

18. In his pocket, Ben has 5 dimes, 6 nickels, and 4 pennies. He selects 3 coins. What is the probability that he selects exactly dime and 1 penny (in no particular order)? 15 total

$6\left(\frac{5}{15} \cdot \frac{4}{14} \cdot \frac{6}{13}\right) = .2637 = \frac{24}{91}$

DPN PND
DNP PDN
NPD
NDP

19. How many ways can 8 charms be arranged on a bracelet **with no clasp**?

OMIT

20. One card is drawn at random from a standard deck. What is the probability of drawing an ace or a red card?

$P(\text{Ace}) + P(\text{Red}) - P(\text{Both})$

$\frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{7}{13}$

21. Of the 17 girls in a classroom, 10 of them have blonde hair. Of the 15 boys in the same classroom, 8 of them have blonde hair. What is the probability of randomly selecting a girl or a student with blonde hair?

$P(G) + P(\text{Blonde}) - P(\text{Both})$

$\frac{17}{32} + \frac{18}{32} - \frac{10}{32} = \frac{25}{32}$

22. Determine if the following is a fair game: Two dice are rolled. If the sum is less than 7, then player A wins \$5 from player B; otherwise, B wins \$4 from A.

(You **MUST** create a table)

	sum <7	sum ≥ 7
\$	5	-4
Prob	$\frac{15}{36}$	$\frac{21}{36}$

$EV = 5\left(\frac{15}{36}\right) - 4\left(\frac{21}{36}\right)$

$EV = -.25$

Not Fair

22. Kay Paso, who is 3 years old, tears the labels off all 10 of the soup cans on her mother's shelf. Her mother knows that there were 2 cans of tomato soup and 8 cans of vegetable soup. She selects 4 cans at random. What is the probability that exactly one of the cans is tomato?

$4\left(\frac{2}{10}\cdot\frac{8}{9}\cdot\frac{7}{8}\cdot\frac{6}{7}\right)=\frac{8}{15}$

TVVV
VTVV
VVTV
VVVT

23. Five cards are dealt from a deck of 52 cards. Which of the following shows the probability that 4 aces will be dealt?

OMIT (a) $\frac{{}_4C_4}{{}_{12}C_5}$ (b) $\frac{{}_{52}C_4}{{}_{52}C_5}$ (c) $\frac{({}_4C_4)({}_{48}C_1)}{{}_{52}C_5}$ (d) $\frac{({}_4C_4)({}_{52}C_1)}{{}_{52}C_5}$

24. Two cards are drawn from a deck of 52 cards with the first card replaced before the second card is drawn. What is the probability that neither card is a spade?

(a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{19}{34}$ $\frac{39}{52}\cdot\frac{39}{52}$

25. A pair of dice is tossed. What is the probability that the sum of the faces showing on top is 10?

(a) $\frac{2}{9}$ (b) $\frac{1}{12}$ (c) $\frac{1}{9}$ (d) $\frac{1}{6}$ $\frac{3}{36}$

26. Suppose you play a game in which you make a bet and then draw a card from a standard deck of 52 cards as well as 2 jokers. If you draw a joker, you keep your bet and win \$5; if you draw a face card, you keep your bet and win \$2; and if you draw any other card, you lose your bet. What is your expected value on this game if you bet \$1?

	Joker	Face	other
\$	6	3	-1
prob	$\frac{2}{54}$	$\frac{12}{54}$	$\frac{40}{54}$

$EV=6\left(\frac{2}{54}\right)+3\left(\frac{12}{54}\right)-1\left(\frac{40}{54}\right)$

$EV=\frac{4}{27}\approx .15$

Solve the following using the binomial probability theorem.

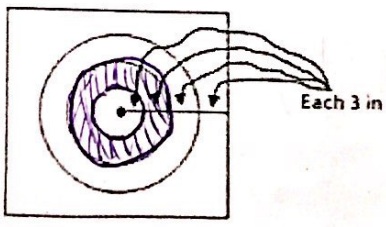
27. What is the probability of getting exactly 2 "fives" in 4 rolls of a die?

$n=4$ $p=\frac{1}{6}$ $r=2$ $\frac{25}{216}$

28. A coin is flipped eight times. Find the probability of getting exactly six heads.

$n=8$ $r=6$ $p=\frac{1}{2}$ $\frac{7}{64}$

29. If you throw a dart at the square board shown, find the probability it will land in the middle ring.



$A_{whole}=576$

$A_{2nd\ circle}=36\pi$

$A_{3rd\ circle}=9\pi$

$=\frac{36\pi-9\pi}{576}$

$=.147$